Field Theory Description of Topological States of Matter

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Outline

- Topological states of matter
- Quantum Hall effect: bulk and edge
- Effective field theory & chiral anomaly
- Ten-fold classification of topological states for non-interacting electrons
- Interacting fermions & anomalies

Topological States of Matter

- System with bulk gap but non-trivial at energies below the gap
- global effects and global degrees of freedom:

→ massless edge states, exchange phases, ground-state degeneracies

- not described by symmetry breaking and Landau-Ginzburg approach
- quantum Hall effect is chiral (B field breaks T symmetry)
- quantum spin Hall effect is non-chiral (T symmetric)
- other systems: QAnomalousHE, Chern Insulators, Topological Insulators and Superconductors, Weyl Semimetals in d=1,2,3
- Ten-fold classification of non-interacting systems (electron bands)

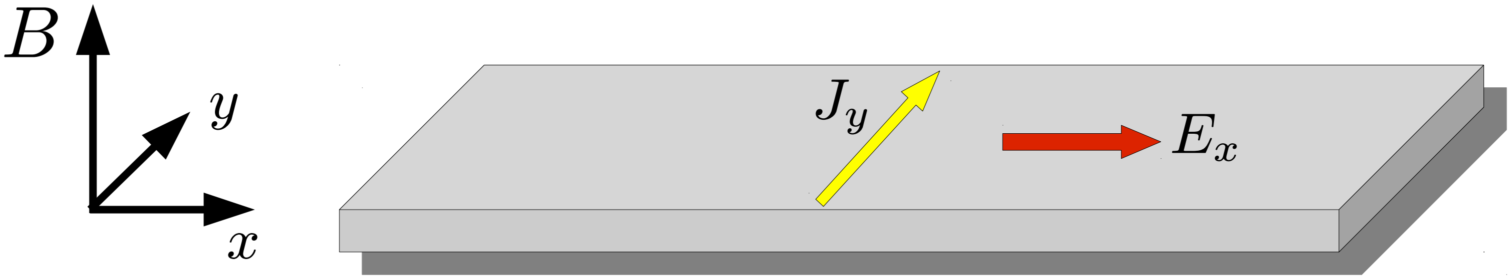
Topological Insulators have been observed in d=2 & 3

(Molenkamp et al. '07; Hasan et al. '08)

Quantum Hall Effect

2 dim electron gas at low temperature T ~ 10-100 mK

and high magnetic field B ~ 5-10 Tesla



Conductance tensor $J_i = \sigma_{ij} E_j, \quad \sigma_{ij} = R_{ij}^{-1}, \qquad i, j = x, y$

Plateaus: $\sigma_{xx} = 0$ no Ohmic conduction → gap

High precision & universality

$$\sigma_{xy} = R_{xy}^{-1} = \frac{e^2}{h}\nu, \quad \nu = 1(\pm 10^{-9}), 2, 3, \ldots \frac{1}{3}, \frac{2}{5}(\pm 10^{-6})$$

Uniform density ground state: $\rho_o = \frac{eB}{hc}\nu$

Incompressible fluid

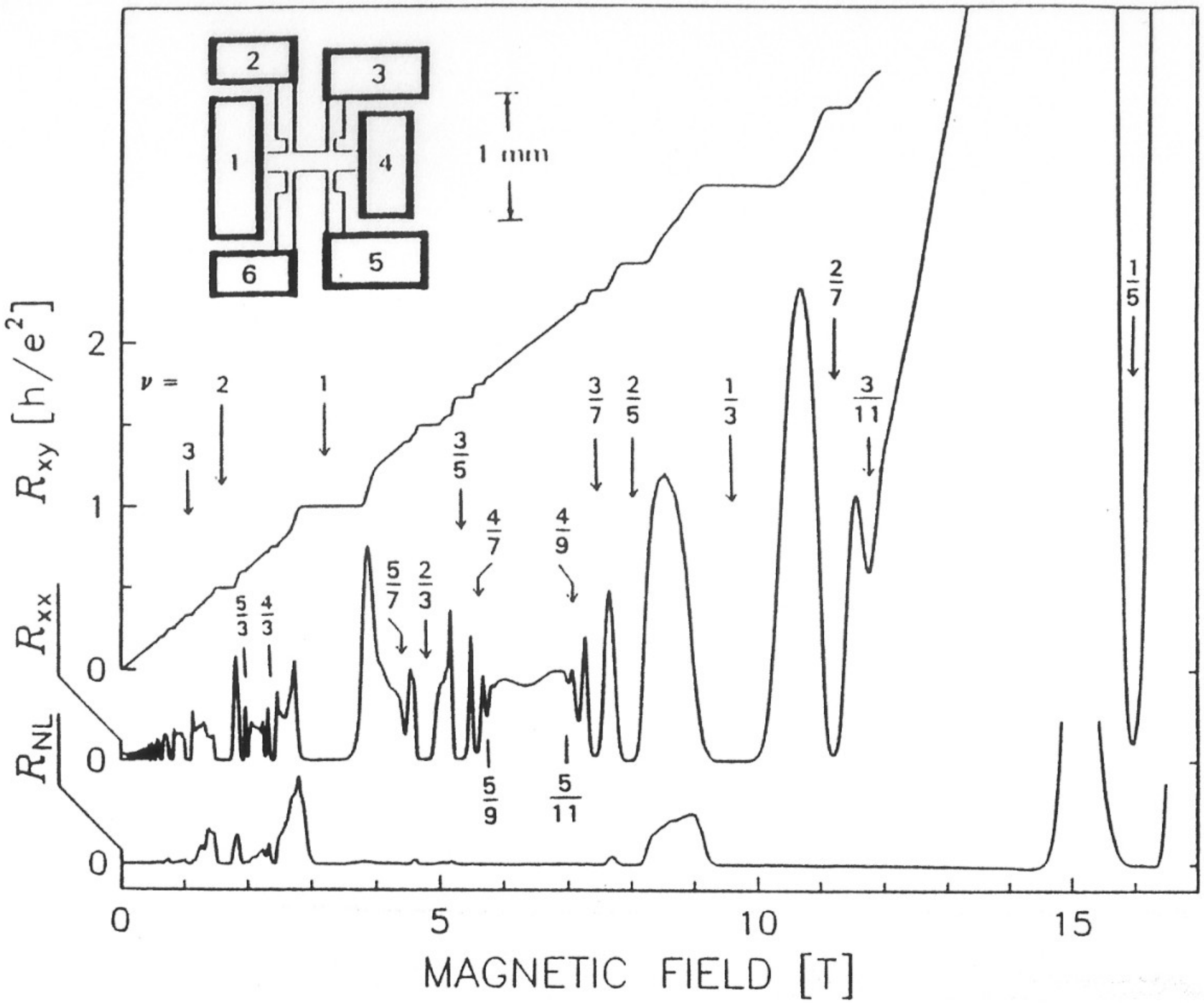
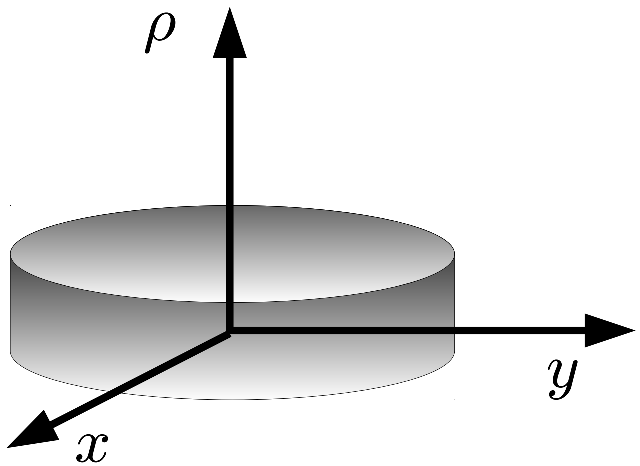


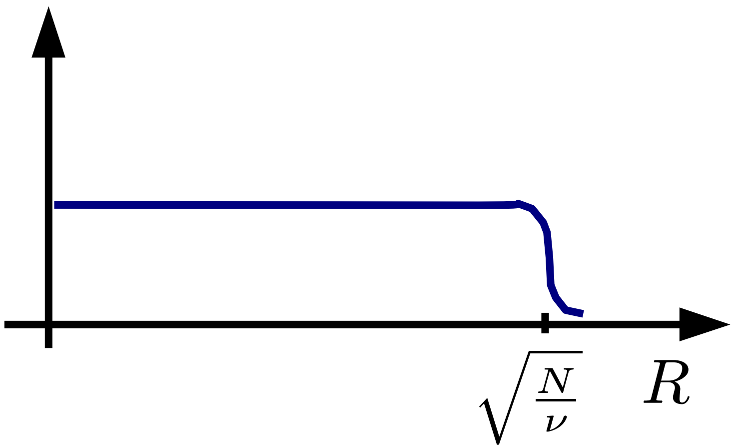
Figure 1. This figure shows a quantum Hall experiment trace. (Source: Goldman *et al.* [2].) The sample geometry is shown in the inset. The various resistances are defined as: Hall resistance $R_{xy} = V_{26}/I_{14}$; longitudinal resistance $R_{xx} = V_{23}/I_{14}$; and non-local resistance $R_{NL} = V_{26}/I_{35}$. Here, V_{jk} denotes the voltage difference between the leads j and k, and I_{jk} denotes the current from lead j to lead k. The experiment was performed at 40 mK.

Laughlin's quantum incompressible fluid

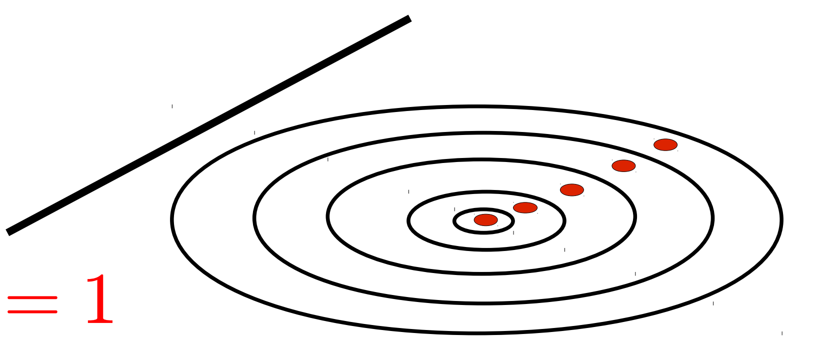
Electrons form a droplet of fluid:

→ incompressible: gap → fluid: $\rho(x,y) = \rho_o = \text{const.}$

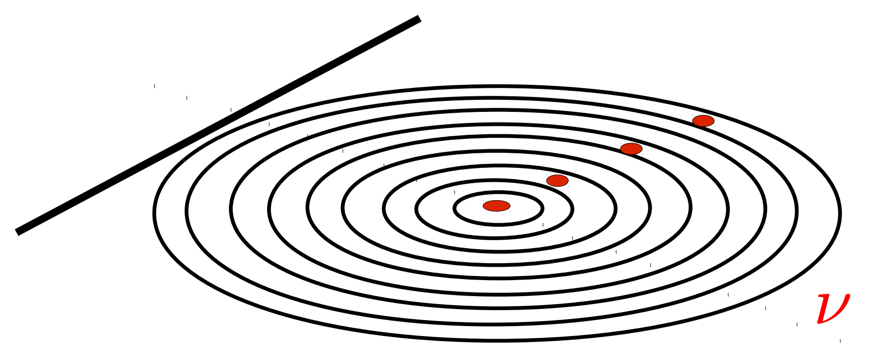




filling fraction: $\nu = \frac{N}{\mathcal{D}_A} = \frac{N}{\Phi/\Phi_o} = 1, 2, \ldots \frac{1}{3}, \frac{1}{5}, \ldots$ $\Phi_o = \frac{hc}{e}$



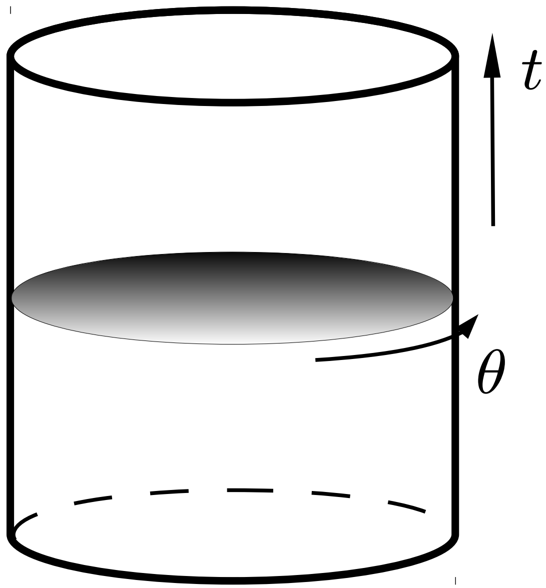
$\nu = 1$

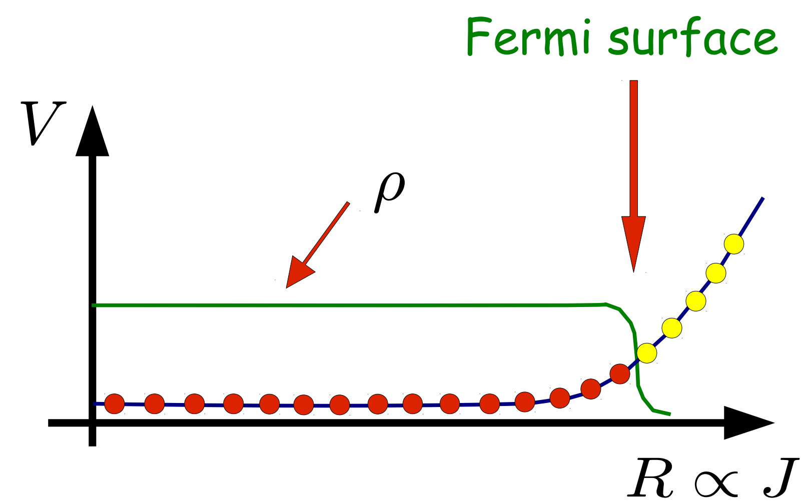


$\nu = \frac{1}{3}$

Edge excitations

The edge of the droplet can fluctuate: massless (1+1)-dimensional edge waves





edge ~ Fermi surface: linearize energy $\varepsilon(k) = \frac{v}{R}(k - k_F), \quad k \in \mathbb{Z}$

relativistic field theory in (1+1) dimensions with chiral excitations (X.G.Wen)

- → conformal field theory of edge excitations (chiral Luttinger liquid)
- → CFT modelling describes nonperturbative quantum effects
- → experimental predictions for conduction and tunneling

Effective field theory

Quantum field theory in a nutshell:

- Take a massive phase and fix a maximal energy scale Λ
- Guess the low-energy degrees of freedom (fields) and symmetries
- Write the action compatible with them, as a power series in the fields and their derivatives ($1/\Lambda$ expansion). Ex. Landau-Ginzburg:

$$S[J] = \int (\partial_\mu \phi)^2 + a\,\phi + b\,\phi^2 + c\,\phi^4 + \cdots + \phi J \qquad a, b, c, \ldots \text{ to be fitted}$$

→ Successful examples: LG, SC, FL, SM, SYM, AdS/CFT, you name it

→ Successful if leading terms are simple: universality

- Topological states need effective theories beyond Landau-Ginzburg, Higgs etc.

→ Topological gauge theories and anomalies

Bulk & boundary

Chern-Simons effective action of QHE

$$S[A] = \frac{\nu}{4\pi}\int dx^3\, \varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho = \frac{\nu}{4\pi}\int AdA$$

Laughlin state $\nu = 1, \frac{1}{3}, \frac{1}{5}, \cdots$

- no local degrees of freedom in (2+1) dim., only global effects

$$\rho = \frac{\delta S}{\delta A_0} = \frac{\nu}{2\pi}\mathcal{B}$$ Density

$$J^i = \frac{\delta S}{\delta A_i} = \frac{\nu}{2\pi}\varepsilon^{ij}\mathcal{E}^j$$ Hall current

- Hall current is topological, i.e. robust
- Introduce Wen's hydrodynamic matter field a_μ and current $j^\mu = \varepsilon^{\mu\nu\rho}\partial_\nu a_\rho$

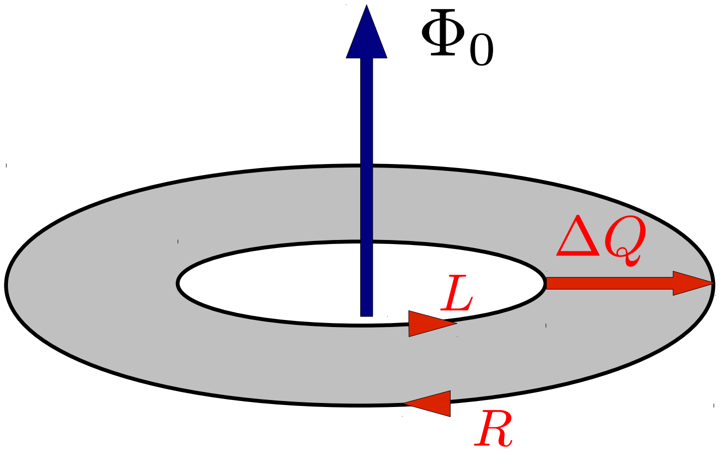
$$S[A] = \int -\frac{\pi}{\nu} ada + A_\mu\, j^\mu = S_{\rm matt}[a] + (\text{e.m. coupl.})$$

- Sources of a_μ field are anyons (Aharonov-Bohm phases $\frac{\theta}{\pi} = \nu = \frac{1}{3}, \cdots$)
- Gauge invariance requires a boundary term in the action:

$$S_{\rm matt}[a] \to S_{\rm matt}[a] + S_{CFT}[\varphi], \quad \partial_\mu \varphi = a_\mu|_b$$ → massless edge states

- Bulk topological theory is tantamount to conformal field theory on boundary

CFT on the boundary and chiral anomaly



- edge states are chiral fermions/bosons
- chiral anomaly: boundary charge is not conserved
- bulk (B) and boundary (b) compensate:

$$\partial_i J^i + \partial_t \rho = 0, \ \rightarrow \ \oint dx J_B + \partial_t Q_b = 0$$

- adiabatic flux insertion (Laughlin)

$$\Phi \rightarrow \Phi + \Phi_0,$$

$$Q_R \rightarrow Q_R + \Delta Q_b = \nu, \qquad \Delta Q_b = \int_{-\infty}^{+\infty} dt \oint dx \ \partial_t \rho_R = \nu \int F_R = \nu \, n$$ chiral anomaly

- Anomaly inflow — Index theorem: exact quantization of Hall current
- edge chiral anomaly = response of topological bulk to e.m. background
- chiral edge states cannot be gapped ⟷ topological phase is stable

Summary

- Quantum Hall effect is a Topological State of Matter:

  - bulk gap and massless edge states
  - electrons non-interacting $\nu = 1$ and interacting $\nu = \frac{1}{3}, \frac{1}{5}, \cdots$
  - effective theory is Chern-Simons (bulk) + CFT (edge)
  - topological and geometrical effects
  - chiral edge states and chiral anomaly

next

- Many other Topological states:

  - non-interacting fermions → band systems → ten-fold classification
  - interacting fermions → anomalies → continuous & discrete

Effective FTs

Ten-fold classification (non interacting)

class\δ	T	C	S	0	1	2	3	4	5	6	7
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
BDI	+	+	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
D	0	+	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
DIII	−	+	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
AII	−	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
CII	−	−	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
C	0	−	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
CI	+	−	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

space dim. d

IQHE → A; Top. Ins. → AII

A, AIII: period 2

AI–CI: period 8 (Bott)

- Study T, C, P symmetries of quadratic fermionic Hamiltonians
- Matches classes of disordered systems/random matrices/Clifford algebras
- Does it extend to interacting systems? YES - NO - ???

(A. Kitaev; Ludwig et al. 09)

→ **study field theory anomalies**

Ten-fold classification

- Fermionic bulk Hamiltonian

$$H = \sum_{i,j} c_i^\dagger M_{ij} c_j$$

classes of matrices M_{ij} depend on C, P, T symmetries and $\mathcal{C}^2 = \pm 1, \mathcal{T}^2 = \pm 1$

⟷ random systems (Altland-Zirnbauer; Cartan symmetric spaces)

- Massless states: fermion bands with level crossing

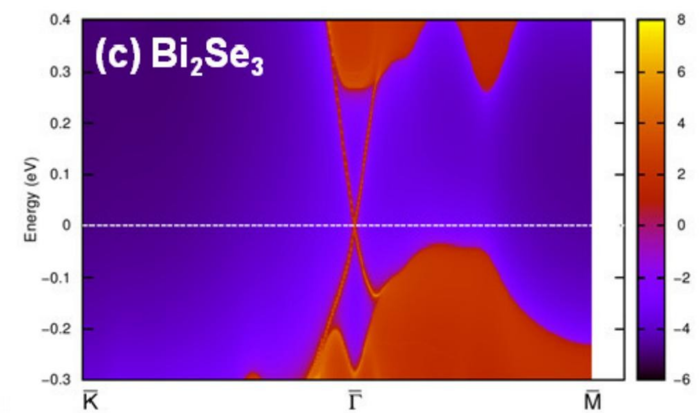
- zoom at low energy

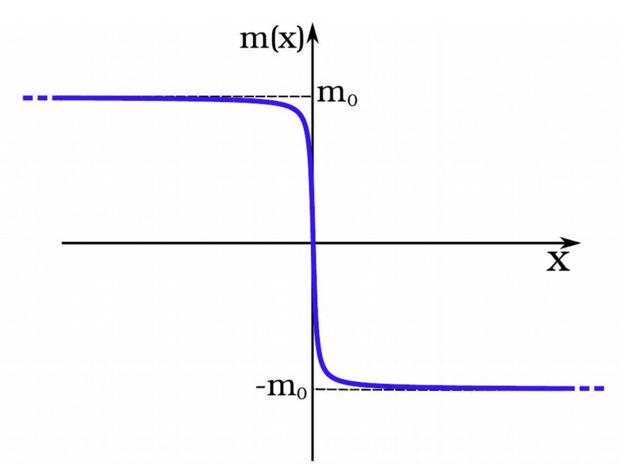
- approximate translation & Lorentz invariance

- massive Dirac fermion with kink mass $m(x)$

- boundary $(d-1)$ dim. massless fermion localized at $x=0$ (Jackiw-Rebbi)

⟷ Clifford algebras





Classification by chiral anomalies: $\mathbb{Z}$ classes

$U(1)$ anomaly: A, AIII

gravitational & mixed anomalies: AI, BDI, D, DIII, AII, CII, C, CI

Cartan\ d	0	1	2	3	4	5	6	7	8	9	10	11	$\cdots$
A	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\cdots$
AIII	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	$\cdots$
AI	$\mathbb{Z}^{\spadesuit}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}^{\spadesuit}$	0	0	0	$\cdots$
BDI	$\mathbb{Z}_2$	$\mathbb{Z}^{\clubsuit}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}^{\clubsuit}$	0	0	$\cdots$
D	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}^{\heartsuit}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}^{\heartsuit}$	0	$\cdots$
DIII	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}^{\diamondsuit}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}^{\diamondsuit}$	$\cdots$
AII	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}^{\spadesuit}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\cdots$
CII	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}^{\clubsuit}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\cdots$
C	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}^{\heartsuit}$	0	0	0	$2\mathbb{Z}$	0	$\cdots$
CI	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}^{\diamondsuit}$	0	0	0	$2\mathbb{Z}$	$\cdots$

- $d = \text{even}$ boundary anomaly, bulk Chern-Simons theory (QHE, A class)
- $d = \text{odd}$ bulk anomaly, bulk theta term, ex. d=3 $U(1)$ gauge theory (AIII)

$$S[A] = \frac{\theta}{32\pi^2}\int F\wedge F = \frac{\theta}{4\pi^2}\int dx^4\, E\cdot B$$

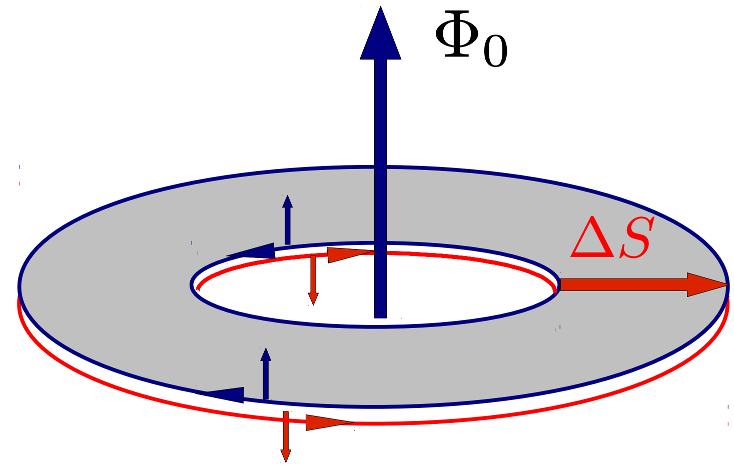
magneto-electric effect

- gravitational anomaly = non-conservation of the thermal current

(A.Ludwig, Furusaki, J. Moore, S. Ryu, Schnyder '08-12)

Non-chiral states & discrete anomalies

Quantum Spin Hall Effect

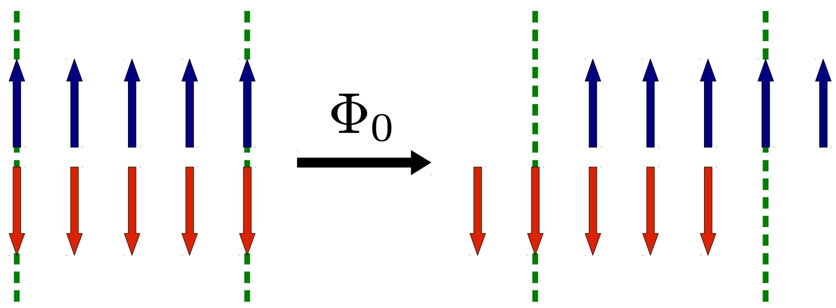


- take two $\nu = 1$ Hall states of spins $\uparrow\downarrow$
- system is Time-reversal invariant:

$$\mathcal{T}: \ \psi_{k\uparrow} \to \psi_{-k\downarrow} \ , \qquad \psi_{k\downarrow} \to -\psi_{-k\uparrow}$$

- non-chiral CFT with $U(1)_Q \times U(1)_S$ symmetry
- adding flux pumps spin → $U(1)_S$ anomaly

(L. Fu, C. Kane, E. Mele 06)



$$\Delta Q = \Delta Q^{\uparrow} + \Delta Q^{\downarrow} = 1 - 1 = 0$$

$$\Delta S = \frac{1}{2} - \left(-\frac{1}{2}\right) = 1$$

- in Topological Insulators $U(1)_S$ is explicitly broken by spin-orbit interaction
- no currents $\sigma_H = \sigma_{sH} = 0$
- yet $\mathcal{T}$ is a good symmetry $\mathcal{T}^2 = (-1)^F$

Topological Insulators (T symmetry)

class\δ	T	C	S	0	1	2	3	4	5	6	7
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
BDI	+	+	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
D	0	+	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
DIII	−	+	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
AII	−	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
CII	−	−	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
C	0	−	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
CI	+	−	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

Top. Ins. → AII

- stability of TI ↔ stability of non-chiral edge states
- T symmetry forbids a mass term for one fermion (for odd numbers)

$$\mathcal{T} : H_{\mathrm{int.}} = m \int \psi_\uparrow^\dagger \psi_\downarrow + h.c. \ \rightarrow \ -H_{\mathrm{int.}}$$

$\mathbb{Z}_2$ classification (free fermions)

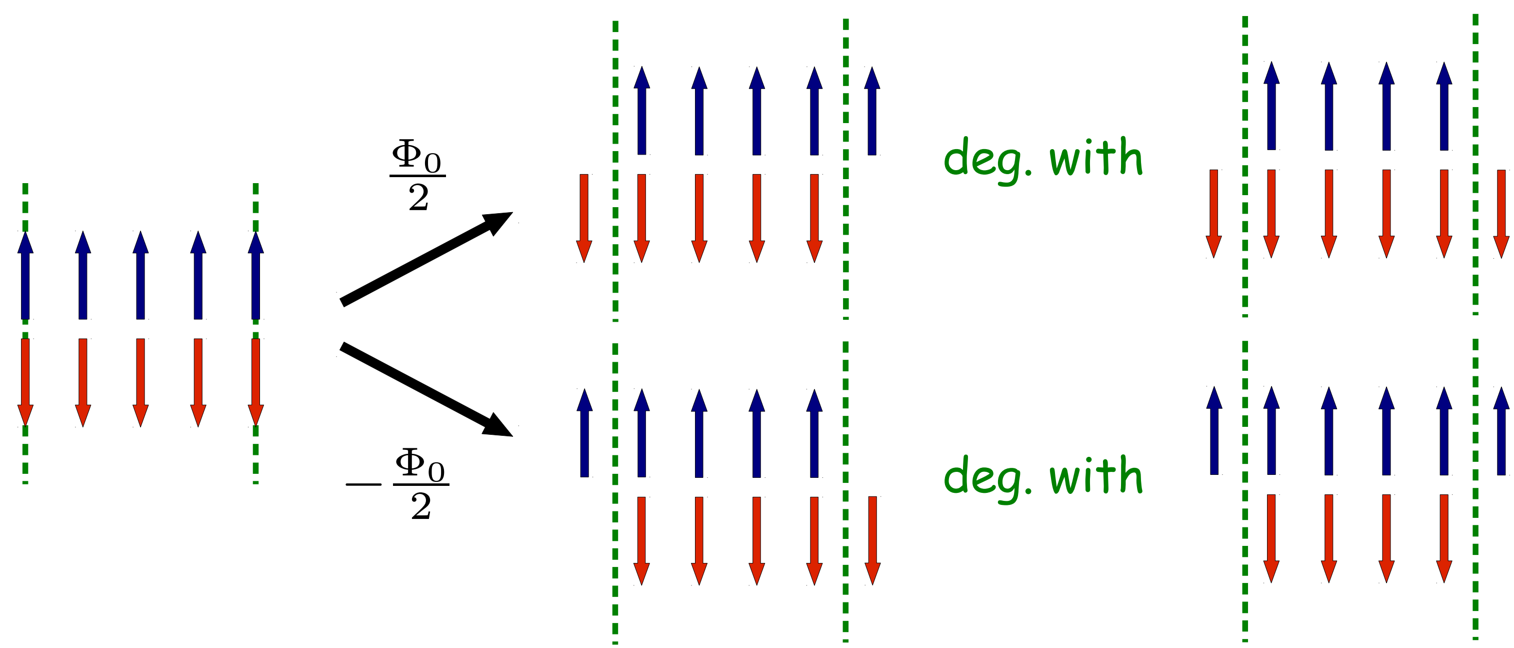
Flux insertion argument

(Fu, Kane, Mele '05-06; Levin, Stern '10-13)

- T symmetry: $\mathcal{T} H[\Phi] \mathcal{T}^{-1} = H[-\Phi]$ & $H[\Phi + \Phi_0] = H[\Phi]$
- T-invariant points: $\Phi = 0, \frac{\Phi_0}{2}, \Phi_0, \frac{3\Phi_0}{2}, \ldots$
- Kane et al. defined a T-invariant $\mathbb{Z}_2$ polarization of band system, equal to the " spin parity" at the edge

$$(-1)^{2S} = (-1)^{N_\uparrow + N_\downarrow}$$

- if $(-1)^{2S} = -1$ there exits a pair of edge states degenerate by Kramers theorem, owing to $\mathcal{T}^2 = -1$

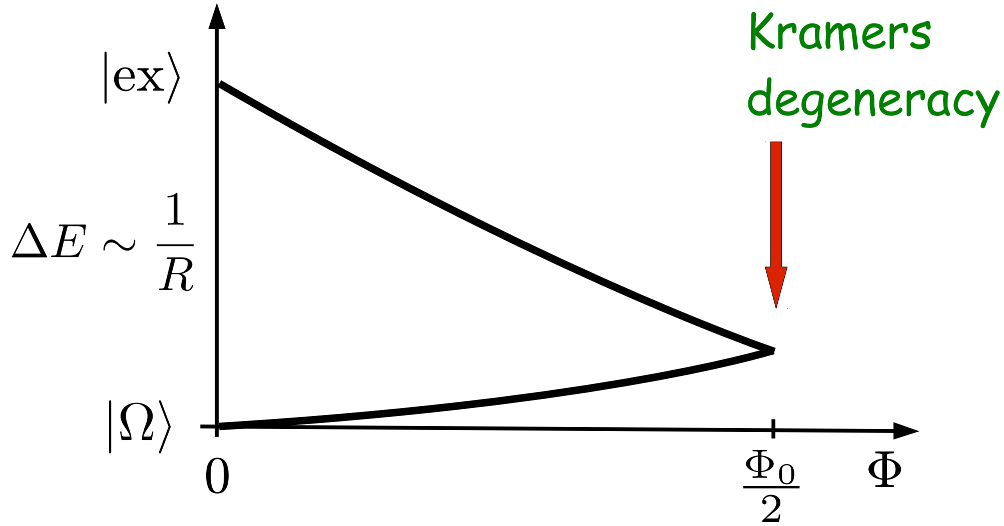


$$\Delta S = \frac{1}{2}$$

$$\Phi = 0: \quad (-1)^{2S}|\Omega\rangle = |\Omega\rangle$$

$$\Phi = \frac{\Phi_0}{2}: \quad (-1)^{2S}|\Omega\rangle = -|\Omega\rangle$$

$|\mathrm{ex}\rangle$ *gapless edge state*



Conclusions

- topological phase is protected by T symmetry (N_f odd)
- spin parity is anomalous, discrete remnant of spin anomaly $U(1)_S \to \mathbb{Z}_2$

> Fu-Kane argument is Laughlin's argument for $\mathbb{Z}_2$ anomaly: $(-1)^{2\Delta S} = -1$

Question: Can we extend this argument to fermions with T-invariant interactions?

Answer: Yes!

→ Fractional topological insulators

$\mathbb{Z}_2$ anomaly in fractional topological insulators

Strategy:

- Study partition functions of TI (& QSHE) using known general results for QHE

(AC, Zemba '97; AC, Viola '10)

- Use them to analyze flux insertions and repeat stability argument

→ $\mathbb{Z}_2$ classification extends to interacting & non-Abelian CFT edges

(AC, Randellini '14 -15)

$(-1)^{2\Delta S} = +1$ unstable

-1 stable

$$2\Delta S = \frac{\sigma_{sH}}{e^*} = \frac{\nu^{\uparrow}}{e^*}$$

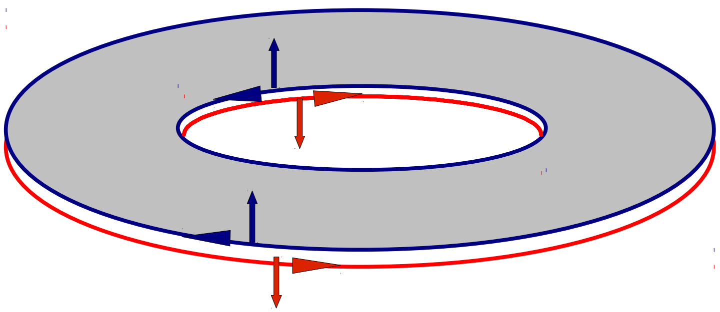
spin-Hall conduct. = chiral Hall conduct. / minimal fractional charge

(Levin, Stern, '09, '12)

- Stability, i.e. $\mathbb{Z}_2$ anomaly, is associated to a discrete gravitational anomaly, i.e. to the lack of modular invariance of the partition function (S. Ryu, S.-C. Zhang '12)

Partition Function of Topological Insulators

- Grand-canonical partition function of
  a single edge, combining the two chiralities



- Four sectors of fermionic systems

$$NS,\ \widetilde{NS},\ R,\ \widetilde{R},\ \text{ risp. }\ (AA),\ \ (AP),\ \ (PA),\ \ (PP)$$

- Neveu-Schwarz sector describes ground state and integer flux insertions:

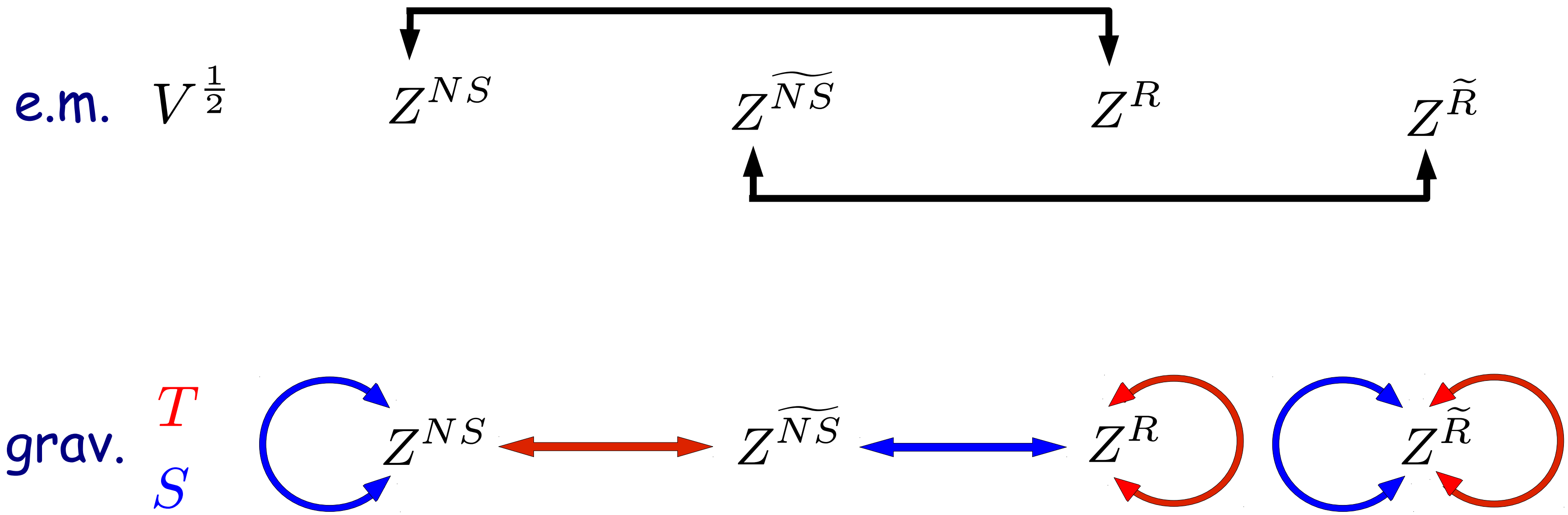
$Z^{NS}(\tau,\zeta) = Z^{NS}(\tau,\zeta+\tau)\,,$ $\quad V : \zeta \to \zeta + \tau$ adds a flux $\Phi \to \Phi + \Phi_0,$

$$\tau = i\beta/L, \qquad \zeta = \beta(iV_o + \mu)$$

- Ramond sector describes half-flux insertions: $\dfrac{\Phi_0}{2}, \dfrac{3\Phi_0}{2}, \ldots$

$$V^{\frac{1}{2}} : Z^{NS}(\tau,\zeta) \ \to\ Z^{NS}\left(\tau,\zeta+\frac{\tau}{2}\right) = Z^{R}(\zeta,\tau)$$

E.m. & gravitational responses



$$V : \zeta \to \zeta + \tau \qquad T : \tau \to \tau + 1 \qquad S : \tau \to -\frac{1}{\tau}$$

Flux addition — Modular transformations

Conclusion

- Many topological states of matter exist and are actively investigated both theoretically and experimentally
- Effective field theories of massless edge excitations and their anomalies describe universal properties and characterize interacting systems
- Theoretical problems both practical and technical:
  - study signatures and observables of topological phases
  - study discrete anomalies (gravitational) in 2d, 3d, K-theory,....

(S. Ryu et al.; G. Moore; E. Witten, N. Seiberg)

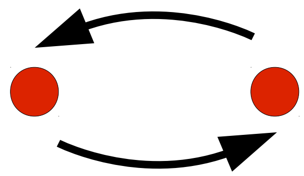
- Technological applications of topologically protected excitations:
  - quantum information and computation
  - conduction without dissipation
  - quantum devices, quantum sensors, etc

Readings

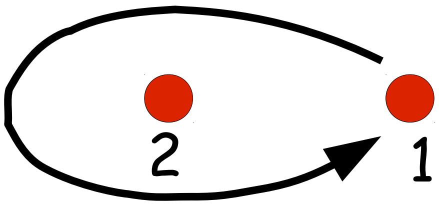
- M. Franz, L. Molenkamp eds., “Topological Insulators”, Elsevier (2013)
- X. L. Qi, S. C. Zhang, Rev. Mod. Phys. 83 (2011) 1057
- C. K. Chiu, J. C. Y. Teo, A. P. Schnyder, S. Ryu, arXiv:1505.03535 (to appear in RMP)

Fractional statistics in 2+1 dimensions

Exchange



Monodromy



$e^{i\theta}$

$$\Psi\left[(z_1 - z_2)e^{i2\pi}, z_2\right] = e^{i2\theta}\Psi\left[z_1, z_2\right]$$

$\theta = \pi\nu,$ e.g. $\nu = 1/3$ fractional $e^{i2\theta} \neq \pm 1$

exchange of identical particles described by the Braid group

$e^{i\theta} \neq e^{-i\theta}$ violates P and T symmetries

If excitations are described by m-dimensional multiplets Ψ_a

$\Psi_a\left[z_1, z_2\right] \;\rightarrow\; U_{ab}\;\Psi_b\left[z_1, z_2\right]$ a,b = 1, . . , m

→ m-dim unitary repres. of braid group

Non-Abelian statistics

Non-Abelian fractional statistics

$\nu = \frac{5}{2}$ described by Moore-Read "Pfaffian state" ~ Ising CFT x boson

Ising fields: I identity, ψ Majorana = electron, σ spin = anyon

fusion rules:

$\psi \cdot \psi = I$ 2 electrons fuse into bosonic bound state

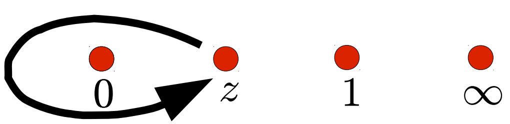
$\sigma \cdot \sigma = I + \psi$ 2 channels of fusion = 2 "conformal blocks"

$\langle \sigma(0)\sigma(z)\sigma(1)\sigma(\infty) \rangle = a_1 F_1(z) + a_2 F_2(z)$ Hypergeometric functions

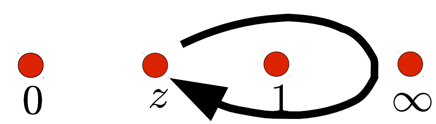
→ state of 4 anyons is two-fold degenerate (Moore, Read '91)

statistics of anyons ~ analytic continuation → 2x2 matrix

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} (ze^{i2\pi}) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} (z)$$



$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix} ((z-1)e^{i2\pi}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} (z)$$



(CFT tech: Verlinde; Moore, Seiberg; Alvarez-Gaume, Gomez, Sierra)

Quantum computation

qubit = two-state quantum system, e.g. spin ½: $|\chi\rangle = \alpha|0\rangle + \beta|1\rangle$

boolean gates → unitary transformations on qubits

→ discrete subgroup of $U(2^n)$ transformations in n qubit Hilbert space

minimal set of generators:

2x2 Pauli matrices + one specific 4x4 matrix

"Universal Quantum Computation"

→ many proposals of systems for QC: excitement & money

quantum computer is unavoidable & useful (e.g. for war, electronic)

big problem: decoherence by the environment

Topological quantum computation

Proposal: use non-Abelian anyons for qubits and operate by braiding (C. Nayak, S. Simon, A. Stern, M.Freedman, S. Das Sarma, 07)

e.g. in Ising-like state $\nu = \frac{5}{2}$

anyons are topologically protected from decoherence (local perturbations):

decay due to finite size $P \sim \exp(-L/\xi), \quad (\text{system} \;\; \text{size})/\ell = O(10^4)$

thermal pair creation $P \sim \exp(-\Delta/T), \quad \Delta/T = O(10^2)$

use 4-spin system $\alpha F_1\rangle + \beta F_2\rangle$ as 1 qubit (2n spin has dim = 2^{n-1})

consider multi-gate bar geometry:

→ perform anyon exchanges by tuning the various gate voltages

Ising is not universal QC; Z_3 parafermions $\nu = \frac{12}{5}$ are OK & others

study other anyonic media, e.g. array of Josephson junctions

many ideas & open problems