Thermal Transport in Hierarchical Hall States

(A.C., M. Huerta, G.R. Zemba, '01)

Outline

- neutral edge modes → thermal current J_Q (Kane, Fisher, '96)
- general formula for thermal conductance

$$K = \frac{\partial J_Q}{\partial T} = \frac{\pi k_B^2 T}{6} (c - \bar{c})$$

gravitational anomaly

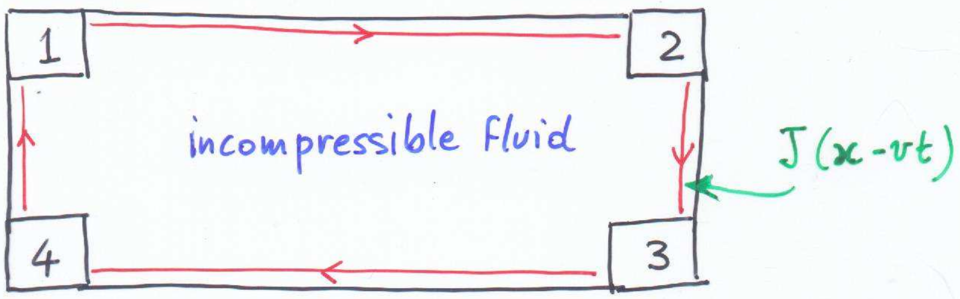
- finite-size corrections to it
- leading corrections are different in the two candidate theories of the hierarchical states:

i) multi-component scalar theory (Luttinger liquid)

vs. ii) $W_{1+\infty}$ minimal model (incompressible fluids)

Thermal transport

Typical setup of conduction experiment



- chiral edge modes carry charge between contacts
- fairly well understood for $\nu = 1, \frac{1}{3}, \frac{1}{5}, \ldots$ (one mode)
- problem: theory of hierarchical edge states

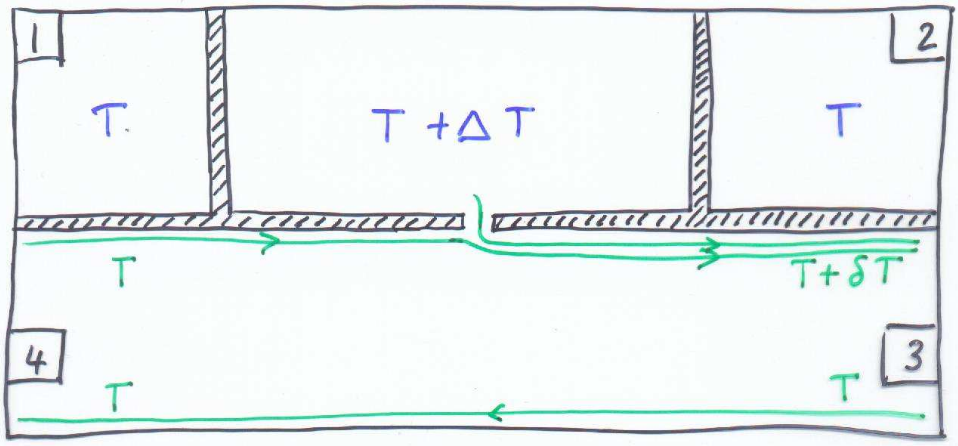
$$\nu = \frac{m}{mp \pm 1}\ , \qquad m = 2, 3, \ldots\ , \qquad p = 2, 4, 6, \ldots$$

predicts $(m-1)$ neutral modes:

HOW TO DETECT THEM ?

• IDEA : measure thermal current

(Kane, Fisher, '95, '96)



• localized heat excess carried away by both charged and neutral modes

• can build a thermometer with a pair of contacts

• local energy balance:

$$\frac{\partial}{\partial t}\mathcal{E}(x,t) + \frac{\partial}{\partial x}\mathcal{P}(x,t) = 0$$

• thermal current

$$J_Q \equiv \langle \mathcal{P} \rangle_T$$

assuming x-independent and steady flow

• thermal conductance

$$K \equiv \frac{\partial J_Q}{\partial T}$$

REMARK: usual way to count degrees of freedom is by the specific heat

$$c_V = \frac{\partial \langle \mathcal{E} \rangle_T}{\partial T}$$

but this is not practical in QHE due to the overwhelming c_V of the ion lattice

Still K and c_V are closely related

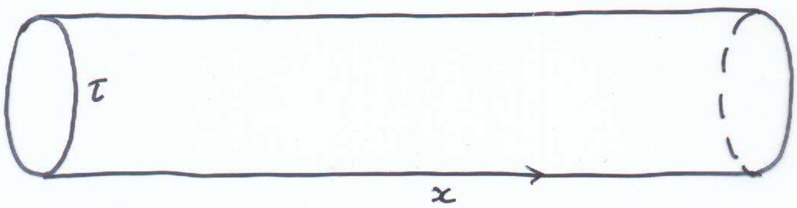
Thermal conductance in CFT

(A.C., M. Huerta, G.R. Zemba)

CFT: general formula for c_V (Affleck; Cardy et al. '86)

IDEA: extend it to K

- Thermal field theory: cylinder geometry



Euclidean time period $\tau \equiv \beta$

- Stress-energy tensor $T(z)$, $\bar{T}(\bar{z})$

$$\mathcal{E} = v\,(T + \bar{T}) \quad , \quad P = v^2 (T - \bar{T}) = J_Q$$

- Map cylinder (w) to plane (z)

$$z(w) = \exp \frac{i 2\pi w}{v\beta} \longleftrightarrow w = v\tau + ix$$

- Expectation value $\langle T(w) \rangle_{\text{cyl.}}$ from anomalous transf. law of T

$$\langle T(w) \rangle - \cancel{\langle T(z) \rangle} \left(\frac{dz}{dw}\right)^2 = \frac{c}{12}\left[\frac{z'''}{z'} - \frac{3}{2}\left(\frac{z''}{z'}\right)^2\right] = \frac{\pi^2 c}{6 v^2 \beta^2}$$

- Thermal current

$$J_Q = \frac{v^2}{L} \int_{-iL/2}^{iL/2} \frac{dw}{2\pi i} \langle T(w) - \bar{T}(\bar{w}) \rangle = \frac{\pi}{12\,\beta^2}(c - \bar{c})$$

- thermal conductance

$$\boxed{K = \frac{\partial J_Q}{\partial T} = \frac{\pi k_B^2 T}{6}(c - \bar{c})}$$

Remarks:

- general & universal result for CFT describing one edge with chiral (c) & anti-chiral ($\bar{c}$) modes.

  Ex: hierarchical edge $\nu = \frac{m}{mp \pm 1}$

  $(+)\ \begin{cases} c = m \\ \bar{c} = 0 \end{cases}$ $\qquad (-)\ \begin{cases} c = 1 \\ \bar{c} = m-1 \end{cases}$

  e.g. $\nu = 2/3 \rightarrow K = 0$

- corresponding result for specific heat (Affleck; Cardy et al)

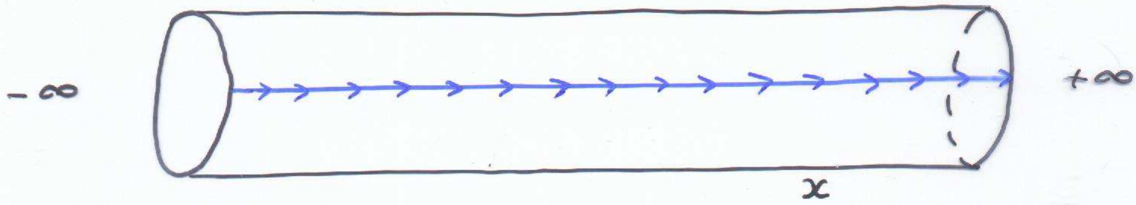
$$c_V = \frac{\pi k_B^2 T}{6 v}(c + \bar{c})$$

thermal conductance		specific heat
• chiral theories $c \neq \bar{c}$	$\longleftrightarrow$	• any theory
• gravitational anomaly		• conformal anomaly
• unusual in stat-mech;		• well known
• sick as string theory		

Anomalies & Non-equilibrium processes

- constant & steady flow implies that energy is conserved locally but not globally: gravitational anomaly equation

$$\nabla^{\bar z} T_{zz} = -\frac{c}{24} \nabla_z R$$



$R \neq 0$ at $x = \pm\infty$, singular points of $z = \exp\left(\frac{x}{\beta}\right)$

- Polyakov, '92:
  - anomalies are violations of conservation laws;
  - anomalous field theories can describe out-of-equilibrium processes;
  - "flux state": constant flux rather than constant quantity.

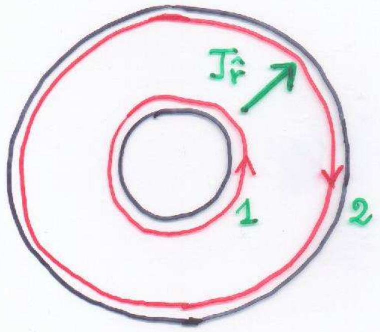
→ QHE is a neat example

- Actually, there are two anomalies: gravitational & chiral

$$\frac{\partial}{\partial \bar{z}} J(z) = \nu \frac{e^2}{2\pi} F \quad , \quad F_{ij} = \varepsilon_{ij} F = \partial_i A_j - \partial_j A_i$$

- take the annulus geometry and integrate over the edge

$$\frac{\partial}{\partial t} Q_1 = \nu \frac{e^2}{2\pi} \int d\theta E_{\hat{\theta}} = -\frac{\partial}{\partial t} Q_2$$



- Hall current

$$J_{\hat{r}} = \sigma_H E_{\hat{\theta}} \quad , \quad \sigma_H = \frac{e^2}{2\pi} \nu$$

- out-of-equilibrium process: spectral flow, i.e. states of definite charge evolve in other states (e.g. electrons are pumped out of the Dirac sea)

Hierarchical Hall States: Two Theories

$$\nu = \frac{m}{mp \pm 1} \quad , \quad m = 2, 3, \ldots \quad , \quad p = 2, 4, 6, \ldots$$

I. m-component scalar theory (Wen, Zee; Read; '91 Fröhlich, Zee; ....)

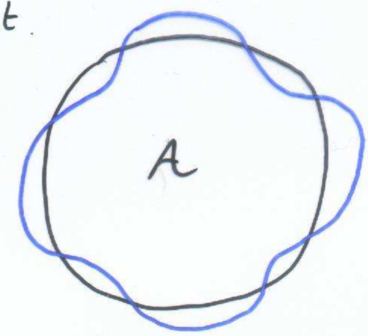
- multi-component generalization of successful theory of edge states of Laughlin's plateaus (chiral Luttinger liquid)

- SU(m) symmetry (m conserved charges; m independent modes; one charged, (m-1) neutral)

II. $W_{1+\infty}$ minimal model (A.C, C. Trugenberger, G. R. Zemba, '95–'01)

- incompressible Hall fluids have natural symmetry under area-preserving coordinate transformations : w_∞ algebra

$$N = \int d^2x\, \rho(x) = \rho_0 \cdot A \rightarrow A = \text{const}$$



- straightforward inplementation of this symmetry in CFT of edge excitations → $W_{1+\infty}$ models (V. Kac et al., '92)

- hierarchical plateaus are in one-to-one correspondence with $W_{1+\infty}$ minimal models, that are similar to theories (I), but have reduced multiplicities of excitations

- SU(m) symmetry is broken;
- m propagating modes, but (m-1) neutral ones are not independent ("interacting");
- numerical evidence of reduced multiplicities in N=10 electron spectrum (cond-mat/9806238)

PROBLEM: Find experimental signatures that distinguish between the two theories

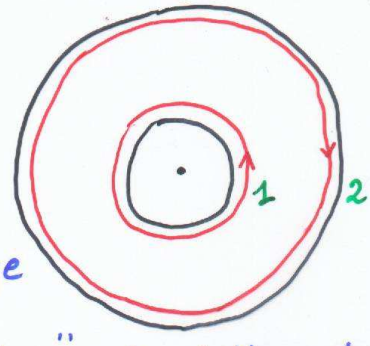
Thermal conductance: $K = \frac{\partial J_Q}{\partial T} = \frac{\pi k_B^2 T}{6} (c - \bar{c})$

- leading order: NO (same $c, \bar{c}$ for I & II)

- first finite-size correction: YES

Finite-size corrections to K

- consider the partition function for the CFT on the annulus



$$Z = \sum_{\lambda=1}^{n} \chi_\lambda^{(1)} \overline{\chi_\lambda^{(2)}}$$

λ = sector of fractional charge

$\chi_\lambda^{(i)}$ = "chiral partition function" of i-th edge

$$\chi_\lambda = \mathrm{Tr}_{(\lambda)} \left\{ q^{L_0 - \frac{c}{24}} \, \bar{q}^{\bar{L}_0 - \frac{\bar{c}}{24}} \right\} \qquad q = e^{2\pi i \tau}$$

$$\tau = \frac{v}{2\pi R}\left(\gamma + i\beta \right)$$

"torsion" $\quad \frac{1}{k_B T}$

R_i = radii

- finite-size expansion:

$$1 \ll v\beta \ll R \qquad 0 < \mathrm{Im}\,\tau \ll 1$$

- express thermal current $\quad \left(\begin{array}{l} J_Q = P = v^2 (T - \bar{T}) \\ \mathcal{E} = v (T + \bar{T}) \end{array} \right)$

$$J_Q^{(1)} = v(\varepsilon - \bar{\varepsilon})^{(1)} = -\frac{i}{2\pi R_1} \left. \frac{\partial \log \chi_\lambda^{(1)}}{\partial \gamma} \right|_{\gamma=0}$$

- expand in τ, or use modular transformation

$$\tau \to \tilde{\tau} = -1/\tau \gg i$$

in the known expressions of χ_λ for both theories I & II (A.C, G.R. Zemba, '97)

• leading $O(1/R)$ correction:

$$K = \frac{\pi k_B^2 T}{6}\left[1 \pm (m-1)\right] + \begin{cases} 0 & \text{multi-component scalar theory} \\ \mp \frac{k_B v}{4\pi R}(m-1) & W_{1+\infty} \text{ minimal model} \end{cases}$$

Remarks:

• leading correction is universal & shape-indep.

• due to reduced multiplicities in minimal models

• $|\Delta K / K| \sim 0.1$ at $T = 50\,mK$

• but leading term can vanish, e.g. $m=2$, $\nu = 2/3$

Further remarks:

• other signatures that are selective between I & II involve dynamics (4-p functions)

• further theories of hierarchical (and paired) Hall states can be analysed similarly (Cristofano, Maiella et al; Pasquier, Serban)