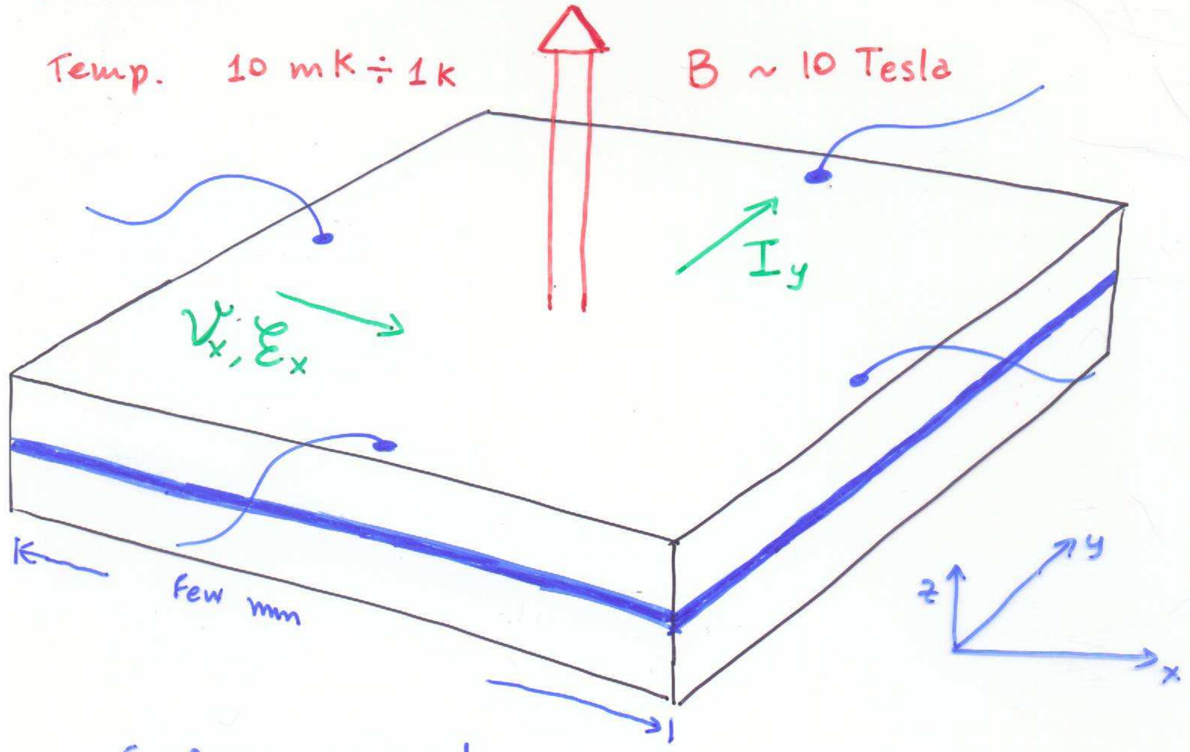
Quantum Hall Effect:

an overview

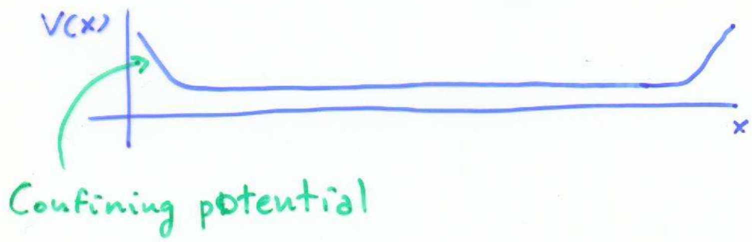
Outline

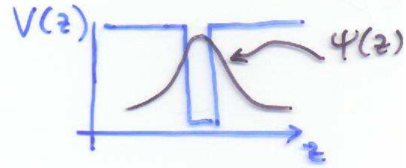
- phenomenology of Integer & Fractional QHE
- Laughlin's theory (incompressible fluid)
  - → anyons $\left(Q = 1/3 \ , \ \frac{\theta}{\pi} = 1/3 \right)$
- edge excitations of Laughlin's states
- Conformal Field Theory description
- experimental confirmations : $Q = 1/3$
- recent activity

TYPICAL EXPERIMENTAL SET UP



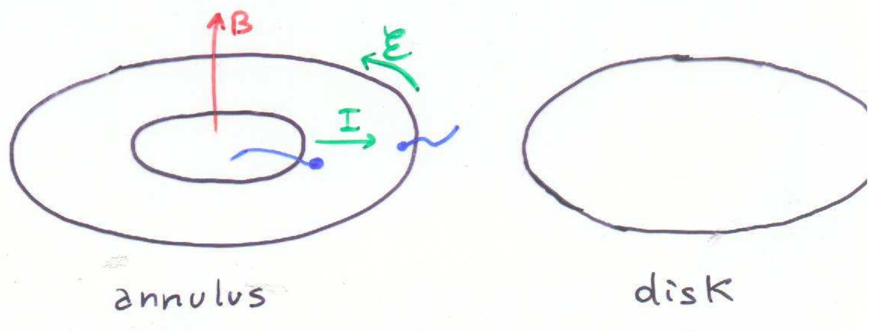
- GaAs compound
- $N \lesssim 10^{11}$ electrons   many-body problem
- two-dimensional problem





- Varying B , measuring $V_i = R_{ij} I_j$, $i,j = x,y$

Equivalent simpler geometries



classical law

$$\underbrace{R_{xy} \propto \frac{1}{\sigma_{xy}} \propto \frac{1}{\nu} \propto \frac{BA}{N}}$$

$$R_{xy}\Big|_{R_{xx}=0} = \frac{1}{\sigma_{xy}}$$

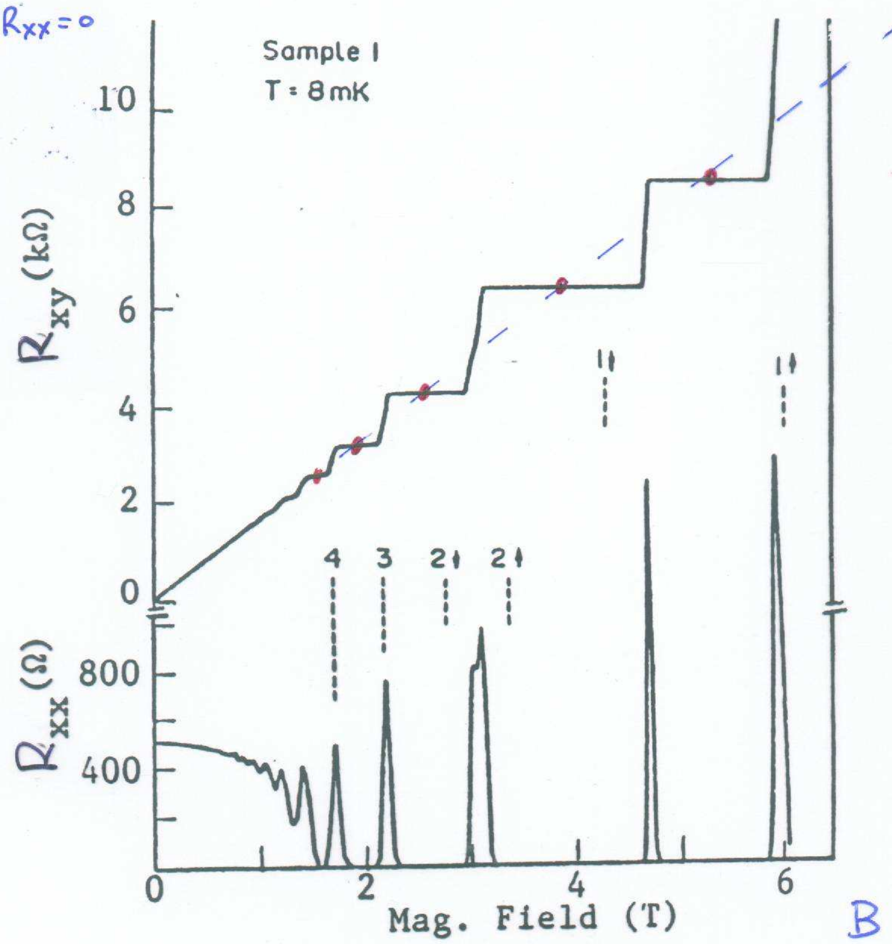


Fig. 1. A sample of the IQHE. *(Ref. 4)* (von Klitzing et al. (1980))

At plateaux:

- $\sigma_{xx} = \underset{\text{gap}}{R_{xx}} = 0$, $R_{xy} = \frac{1}{\sigma_{xy}}$
- $\sigma_{xy} = \frac{e^2}{h}\nu$, $\nu = 1, 2, 3, \ldots \; (\pm 10^{-8})$

very stable universal values

$$V_i = R_{ij} I_j \quad , \quad i,j = x,y \quad \text{or} \quad J_i = \sigma_{ij} \mathcal{E}_j$$

$$R_{ij} = \begin{pmatrix} R_{xx} & R_{xy} \\ R_{xy} & R_{xx} \end{pmatrix} = \begin{pmatrix} 0 & R_{xy} \\ R_{xy} & 0 \end{pmatrix}$$ at centers of plateaux

$$\sigma_{ij} \propto R_{ij}^{-1} = \begin{pmatrix} 0 & \sigma_{xy} \propto R_{xy}^{-1} \\ \sigma_{xy} & 0 \end{pmatrix}$$

Experimental results:

- $\sigma_{xx} = 0$ No Ohmic conduction $\to$ Gap
- $\sigma_{xy} = \frac{e^2}{h} \nu$, $\nu = 1 (\pm 10^{-8}), 2, 3, \ldots, \frac{1}{3} (\pm 10^{-6}), \ldots$

  high precision: very stable ground state with uniform density

  $$\rho(x,y) \cong \rho_0 = \frac{eB}{hc} \nu$$

- universality
- non-trivial pattern of fractional values of ν: Fractional Hall effect

Landau levels : one-electron states

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} - e\vec{A}\right)^2 , \qquad A = \frac{B}{2}(-y, x) , \qquad z = x + iy$$

$$\partial = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)$$

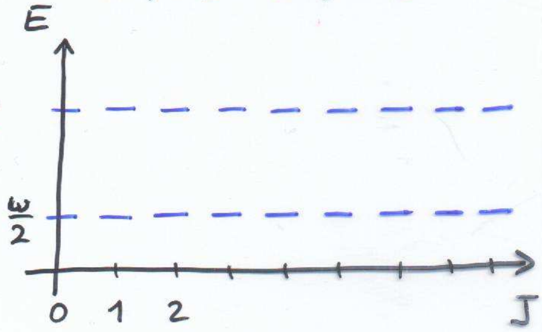
$$\begin{cases} c = \dfrac{z}{2\ell} + \ell\bar{\partial} \\ c^{+} = \dfrac{\bar{z}}{2\ell} - \ell\partial \end{cases} \qquad \begin{cases} b = \dfrac{\bar{z}}{2\ell} + \ell\partial \\ b^{+} = \dfrac{z}{2\ell} - \ell\partial \end{cases}$$

$$[c, c^{+}] = 1 , \quad [b, b^{+}] = 1$$

$$[c, b] = [c, b^{+}] = 0$$

$$\mathcal{H} = \hbar\omega\left(c^{+}c + \frac{1}{2}\right) \qquad \omega = \frac{eB}{mc}$$

$$J = \vec{x} \wedge \vec{p} = b^{+}b - c^{+}c$$



magnetic length $\ell = \sqrt{\dfrac{2\hbar c}{eB}}$ minimal orbit $\approx$ unit flux

$\ell = 100 \div 1000$ Å $\pi\ell^2 B = \phi_0 = \dfrac{hc}{e}$

- **Lowest Landau level:**

$$0 = c\,\varphi_{0,j}(z, \bar{z}) = \left(\ell\bar{\partial} + \frac{z}{2\ell}\right)\varphi_{0,j} , \qquad \varphi_{0,j} = e^{-\frac{1}{2}\frac{|z|^2}{\ell^2}} \left(\frac{z}{\ell}\right)^{j} \frac{1}{\sqrt{j!}}$$

$J\varphi_{0,j} = j\,\varphi_{0,j}$ orbitals peaked at $|z|^2 = \ell j$

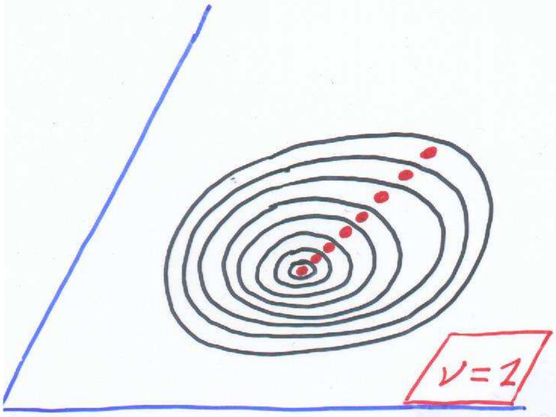
- degeneracy D_A = # of states in a given area A

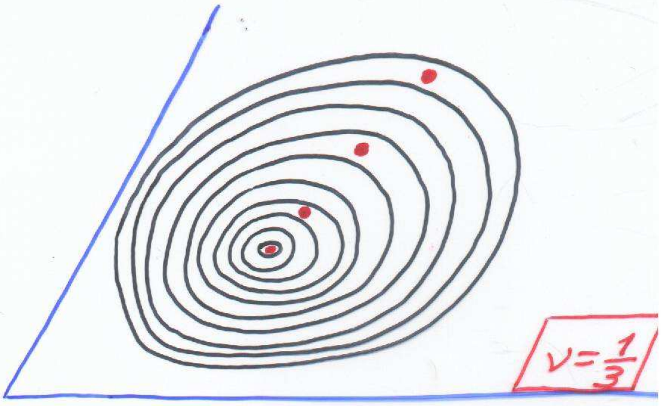
$$D_A = \frac{BA}{\phi_0} = \frac{\phi}{\phi_0} = \#\text{ fluxes}$$

- filling fraction

$$\nu = \frac{N_{\text{electrons}}}{D_A} = \frac{N}{BA/\phi_0} = \frac{\overset{\text{uniform}}{\rho_0}}{B}\,\frac{hc}{e}$$

$$\nu = \frac{\#\ \text{electrons}}{\#\ \text{states}} = \frac{N}{BA/\Phi_0}$$ (density of the QM problem)





+ other fillings of same ρ_0

- finite sample : B controls the filling ν
- infinite plane : add confining potential

$$\mathcal{H} \to \mathcal{H} + \alpha J \qquad \langle J \rangle \approx \frac{N^2}{2}\frac{1}{\nu}$$

- Lowest Landau level = Bargman-Fock space

$$\epsilon_j = \cancel{\frac{\omega}{2}} + \alpha j \qquad (\text{length } \ell = 1)$$

$$\varphi(z,\bar z) = e^{-\frac{1}{2}|z|^2}\chi(z) \qquad \chi \text{ analytic, entire}$$

$$\langle \chi | \varphi \rangle = \int d^2z\, e^{-z\bar z}\, \overline{\chi(z)}\, \varphi(z)$$

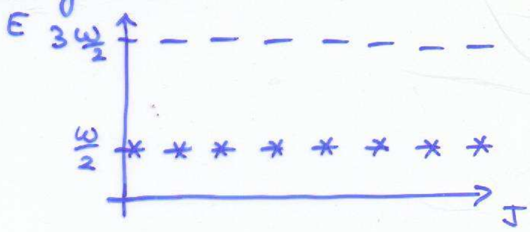
$$\langle z\chi | \varphi \rangle = \int d^2z\, e^{-z\bar z}\, \bar\chi\, \bar z\, \varphi = \int d^2z\, e^{-z\bar z}\, \bar\chi\, (\partial\varphi) = \langle \chi | \partial\varphi \rangle$$

$\bar z$ represented by $\frac{\partial}{\partial z}$, $[\bar z, z] = 1$ quantum plane

Laughlin's incompressible fluid

It is exemplified by the ground state at $\nu=1$

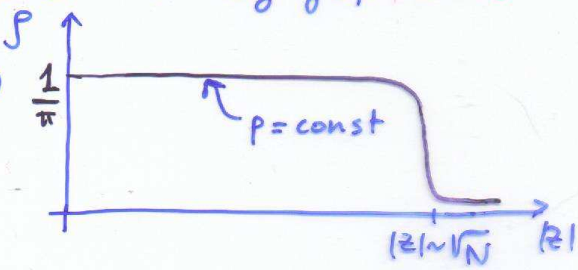
$$\Psi_{g.s}(z_1 \cdots z_N) = \prod_{i<j=1}^{N} (z_i - z_j)\, e^{-\frac{1}{2}\sum_i |z_i|^2}$$



This is incompressible: compressions → lower J → electrons to the next level → big gap $\omega \propto B$

The density profile ($\ell=1$)

$$\rho(z) = \frac{1}{\pi} e^{-|z|^2} \sum_{j=0}^{N-1} \frac{|z|^{2j}}{j!}$$



Electrons behave as a droplet of liquid without phonons

Incompressible fluid

gap $\quad\quad$ $\rho = const$

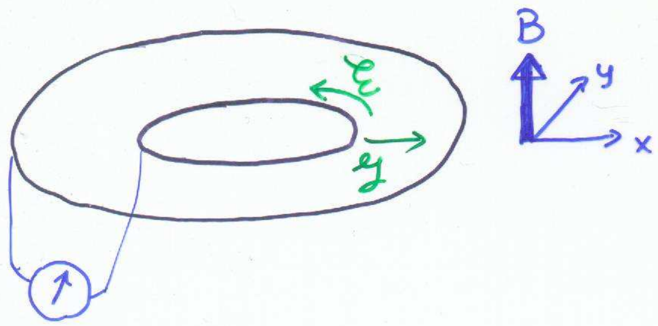
Thus the Hall conduction is given by a rigid translation of the droplet, and is given by naive formulas

$$\nu = \frac{\rho_0}{B}\frac{hc}{e}\,, \quad \frac{V_x}{c} B = \mathcal{E}_y \quad \text{(Lorentz force)}$$

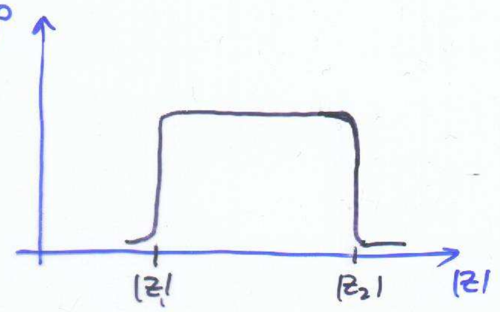
$$J_x = e\, v_x\, \rho_0 = e \cdot \frac{\mathcal{E}_y c}{B} \cdot \frac{B e \nu}{hc} = \frac{e^2}{h} \nu\, \mathcal{E}_y = \sigma_{xy}\, \mathcal{E}_y$$

$$\sigma_{xy} = \frac{e^2}{hc}\nu \quad (\text{for } \nu = 1, 2, 3, \ldots)$$

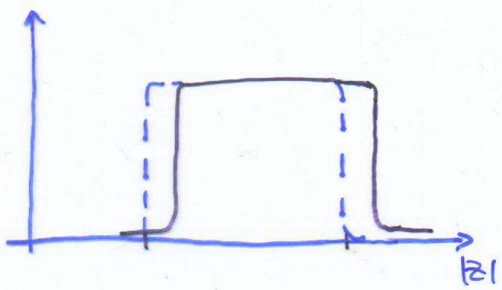
i.e. $\quad \sigma_{xy} \propto \nu \propto \rho_0$ (in fundamental units)

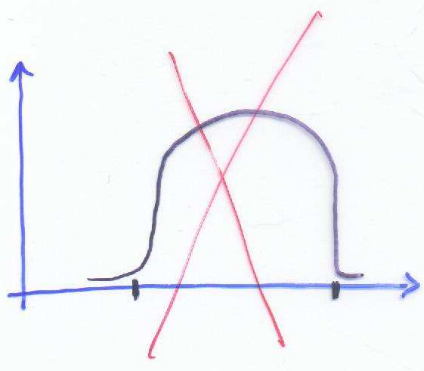


$\mathcal{E} = 0$



$\mathcal{E} \neq 0$





Fractional Hall Effect

$$\sigma_{xy} = \frac{e^2}{hc}\nu \quad , \quad \nu = \frac{1}{3}, \frac{1}{5}, \dots, \frac{2}{5}, \frac{2}{7}, \frac{3}{5}, \dots \ (\pm 10^{-6})$$

(Tsui, Stormer, Gossard (1982))

- less impurities (higher mobility of electrons)
- lower temperatures & higher magnetic fields

gap $\Delta \sim 10^{-2} \frac{e^2}{\kappa \ell} < 10^{-2} \hbar\omega_c$ $(\Delta \sim 10^{-4} eV \sim 1K)$

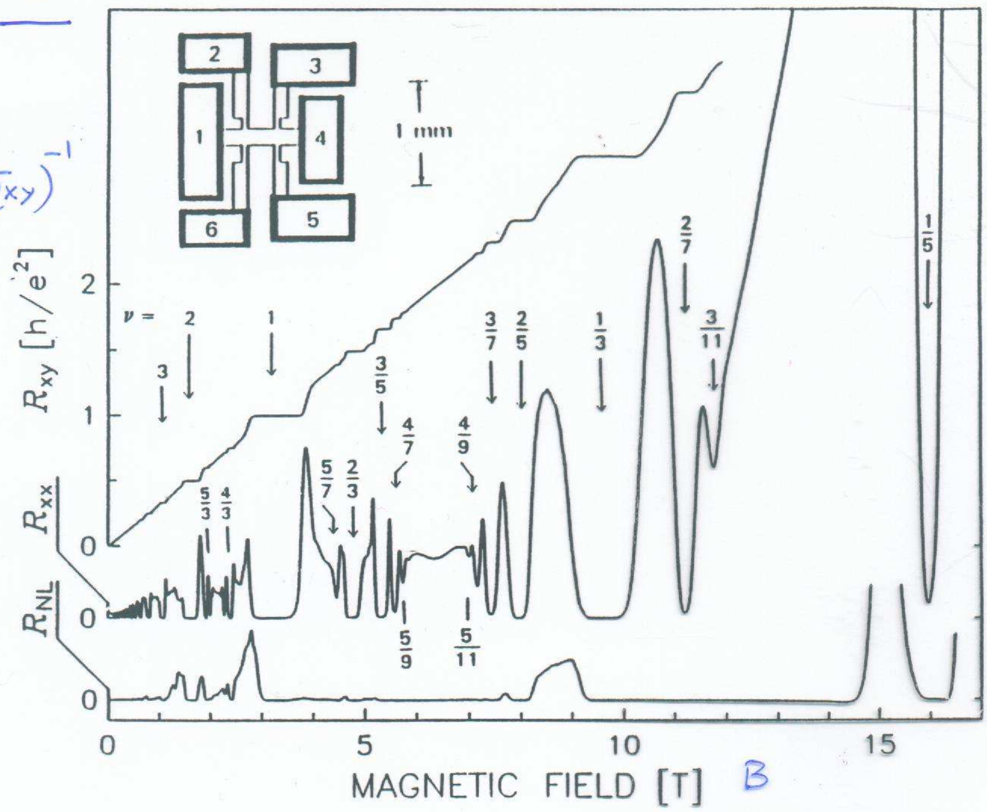
- again universality
- precision limited by experimental apparatus

$\rightarrow$ again exactness

- stable gapful ground state with fractional filling : non-trivial many-body interaction lifts the huge degeneracy of free electrons
- exact fractions suggest that the dynamics is dominated by symmetries

B, y, x, E_i

$R_{xy} = (\sigma_{xy})^{-1}$



Incompressible Fluid at $\nu = \frac{1}{m}$, $m = 2k+1$

Free electrons are highly degenerate → need repulsive Coulomb interaction → non-perturbative

→ Laughlin's trial wave function

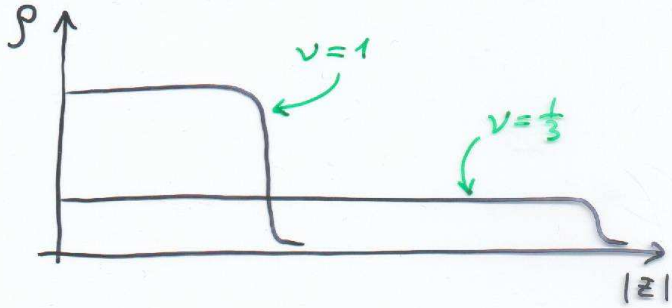
$$\Psi_{\nu=\frac{1}{2k+1}} = \prod_{i<j}(z_i - z_j)^{2k}\,\Psi_{\nu=1} = \prod_{i<j}(z_i-z_j)^{2k+1}\, e^{-\frac{1}{2}\sum|z_i|^2}$$

- unique factorized w.f. at $\nu = \frac{1}{2k+1}$ (higher zero)
- perfect variational guess, almost exact!
- repelling electrons are in a "symmetric" configuration

→ gap, constant density

Incompressible fluid

→ $\sigma_{xy} = \frac{e^2}{h}\,\frac{1}{2k+1}$



Laughlin argued the fluid character ($\rho = \text{const.}$) and the gap from the "plasma analogy":

$\rho_{\frac{1}{2k+1}}(z,\bar{z}) \approx$ density of a two-dimensional plasma

$$Z_{plasma} = \|\Psi_{\frac{1}{2k+1}}\|^2 = \int \prod d^2 z_i \; e^{-\beta \mathcal{H}_{plasma}}$$

$$\mathcal{H}_{plasma} = m \sum_i |z_i|^2 - m^2 \sum_{i<j} \log|z_i - z_j|^2 \qquad \left(\beta = \tfrac{1}{m}\right)$$

background $\Delta|z|^2 = 4 = \text{cost}$

"charge" $= m$

2-d Coulomb

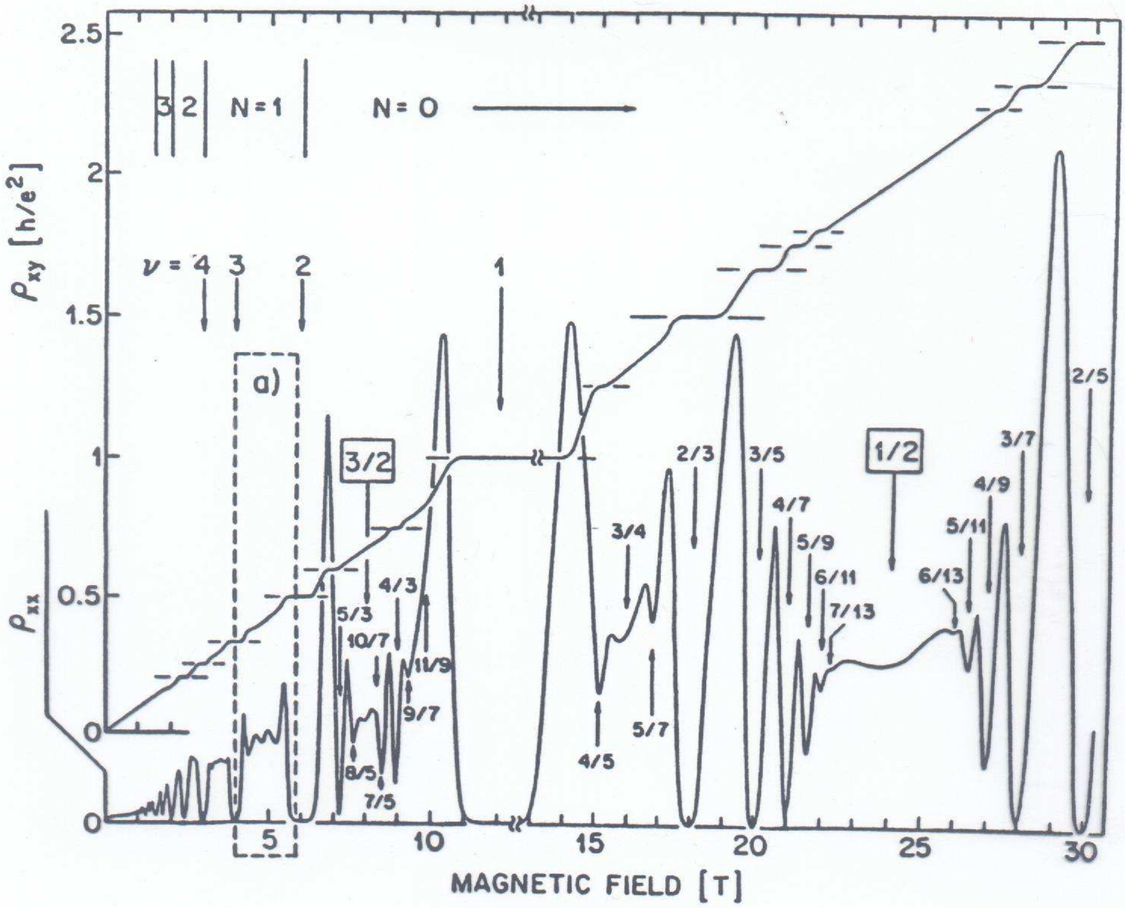


Fig. 3. Overview of both IQHE and FQHE. *(Ref. 6)*

Jain hierarchy

$\left(\frac{1}{2}\right)^{+} \qquad \nu = \frac{n}{2n+1} = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}, \cdots$

$\left(\frac{1}{2}\right)^{-} \qquad \nu = \frac{n}{2n-1} = \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \frac{6}{11}, \frac{7}{13}, \frac{8}{15}, \cdots$

For $m \ll 100$, the plasma is a liquid, charges are screened and local electrical neutrality yields $\rho = \text{const}$ (semiclassical analysis, numerical simulations)

→ use this analogy to find the properties of the excitations of this incompressible fluid (and the analogy with the superfluid)

Gapful excitations of Laughlin's state

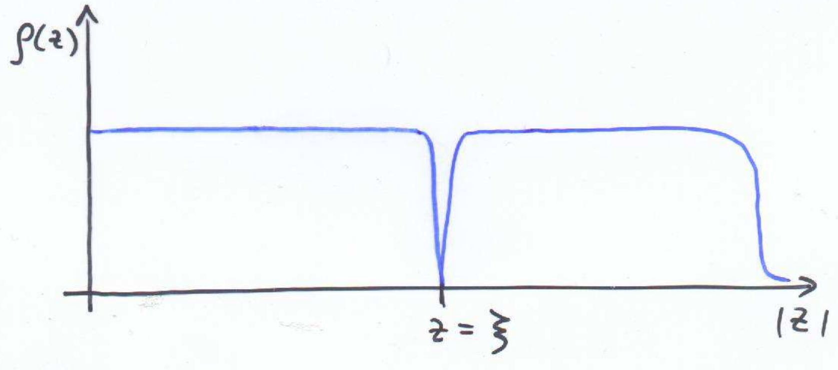
- density waves: violate incompressibility; high gap $O(\omega_c) \gg \Delta \to$ neglected

- quasi-hole = vortex

Ex. $\nu = 1$: remove one electron at $z = \xi$

$$\Psi(\xi; z_1, \dots, z_N) = \prod_{i=1}^{N} (\xi - z_i) \prod_{i<j}^{N} (z_i - z_j) \, e^{-\frac{1}{2}\sum |z_i|^2}$$

like g.s. of $(N+1)$ electrons, not integrating over one



- Remove one electron from Laughlin's state (exact)

$$\Psi(\xi; z_1, \ldots, z_N) = \prod_{i=1}^{N} (\xi - z_i)^m \prod_{i<j} (z_i - z_j)^m \, e^{-\frac{1}{2}\sum |z_i|^2}$$

- Remove one quasi-particle (ansatz) → quasi-hole

$$\Psi(\xi; z_1, \ldots, z_N) = \prod_{i=1}^{N} (\xi - z_i) \prod_{i<j=1}^{N} (z_i - z_j)^m \, e^{-\frac{1}{2}\sum |z_i|^2}$$

- Estimate the charge of the quasi-hole: think to the 2-D plasma analogy

$$\|\Psi\|^2 = \int \pi d^2 z_i \, \exp -\frac{1}{m} \Big\{ m \sum |z_i|^2 - m^2 \sum_{i<j} \log |z_i - z_j|^2 - m \sum_i \log |z_i - \xi| \Big\}$$

other charges see $1/m$ of the electron charge at $z = \xi$

$$Q_{q\text{-}h} = \frac{e}{m} \qquad \text{at } \nu = \frac{1}{m} \qquad \text{Fractional charge}$$

Observed experimentally!! (Glattli et al; 1997
De Picciotto et al; 1997)

Shot-noise experiment: CFT, discussed later on;
1998 Nobel prize to Laughlin, Tsui, Stormer

- quasi-particle charge is exact for many reasons: here because the 2D plasma screens perfectly

- quasi-hole size (vortex core) is $\sim \ell$ (m-indep)

$$Q = \int_{\text{Disc}} \rho \sim \pi \ell^2 \cdot \frac{1}{\pi \ell^2} \frac{1}{m}$$

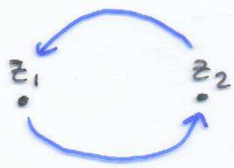
- quasi-hole gap $\sim$ Coulomb self-energy of the disc with uniform charge density $\frac{1}{m}$

$$\Delta \sim \frac{1}{m^2} \frac{e^2}{\kappa \ell} \sim 10^{-1} \div 10^{-2} \frac{e^2}{\kappa \ell} \quad \text{(ok exp.)}$$

Fractional statistics

- Exchange of two electrons

(= ½ of monodromy transformat.)



analytic continuation $(z_1 - z_2) \to e^{i\theta}(z_1 - z_2) \quad \theta \in [0, \pi]$

$$\Psi \sim \prod_{i<j} (z_i - z_j)^m \longrightarrow e^{i\pi m} \Psi = -\Psi \quad \text{(Fermi statistics)}$$

- exchange of two quasi-holes:

$$\Psi_{2q\text{-}h}(\xi_1, \xi_2; z_1, \dots, z_N) = (\xi_1 - \xi_2)^{\frac{1}{m}} \prod_{i=1}^{N} (\xi_1 - z_i)(\xi_2 - z_i) \prod_{i<j} (z_i - z_j)^m$$

extra factor is produced by the plasma to ensure complete screening $\sum_{i<j} q_i q_j \log |x_i - x_j|$

$x_i \leftarrow z_i, \xi_j$

$$\Psi_{2q\text{-}h} \longrightarrow e^{i\frac{\pi}{m}} \Psi_{2q\text{-}h}$$

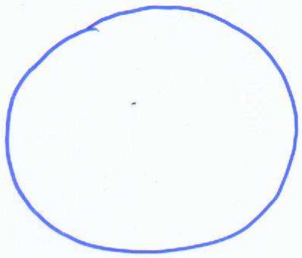
Fractiona statistics $\frac{\theta}{\pi} = \frac{1}{m} = \frac{1}{3}, \frac{1}{5}, \dots$

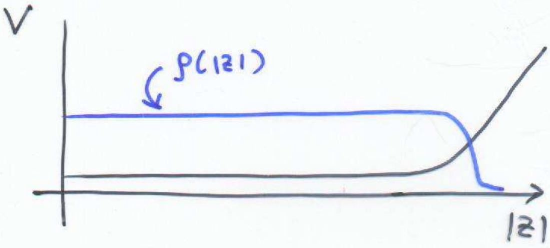
quasi-hole is an <u>anyon</u> (Wilczek et al)

- fractional statistics is a long-range correlation of vortices, independent of distance, i.e. "topological"
  (nicely modelled by conformal field theory or Chern-Simons gauge field)

Edge excitations of the incompressible fluid

The sample has a boundary, with a confining potential; take e.g. a disk:



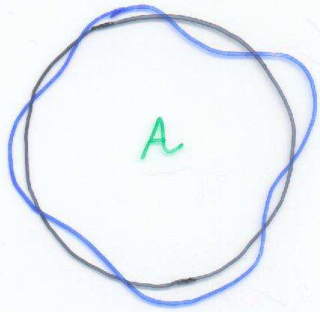


The incompressible fluid satisfies $\rho(|z|) = \rho_0$, i.e. (2+1)-dimensional waves have a high gap and can be neglected.

But:

- the boundary shape can fluctuate:

→ "neutral" edge excitations



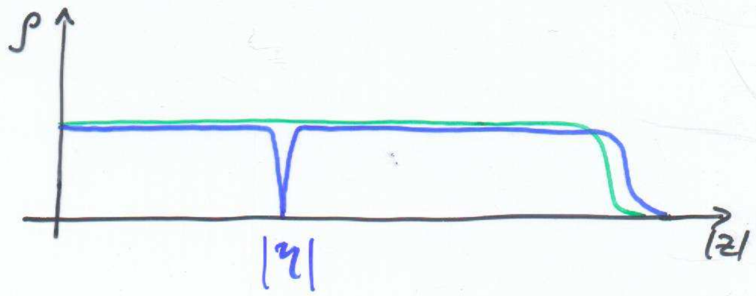
- almost gapless
- excitations satisfy A = cost

$$N = \int d^2x \, \rho(x) = \rho_0 \cdot A \quad , \quad N, \rho_0 = \text{cost}$$

→ area-preserving diffeomorphisms of the plane = W_∞ symmetry (my work with C. Trugenberger & G. Zemba (92))

- $\nu = 1$ obvious: the filled Landau level is like a Fermi sea; edge excitations = particle-hole excitations at the Fermi surface

- charged quasi-hole excitation also observed at the edge



depleted density is spilled at the edge
$\rightarrow$ charged excitation at the edge

Conclusion: both excitations of the incompressible fluid can be detected at the edge

Idea: conformal field theory description of edge excitations $(\xi^1, \xi^2) = (t, R\theta)$

chiral Luttinger theory $(c=1, \bar{c}=0)$

Main features

- effective description valid at low energy, it does explain the conduction experiments

- spectrum at $\nu = 1/3$:

$$Q = \frac{n}{3} \qquad n = \pm 1, \pm 2, \dots \qquad \text{fractional charge}$$

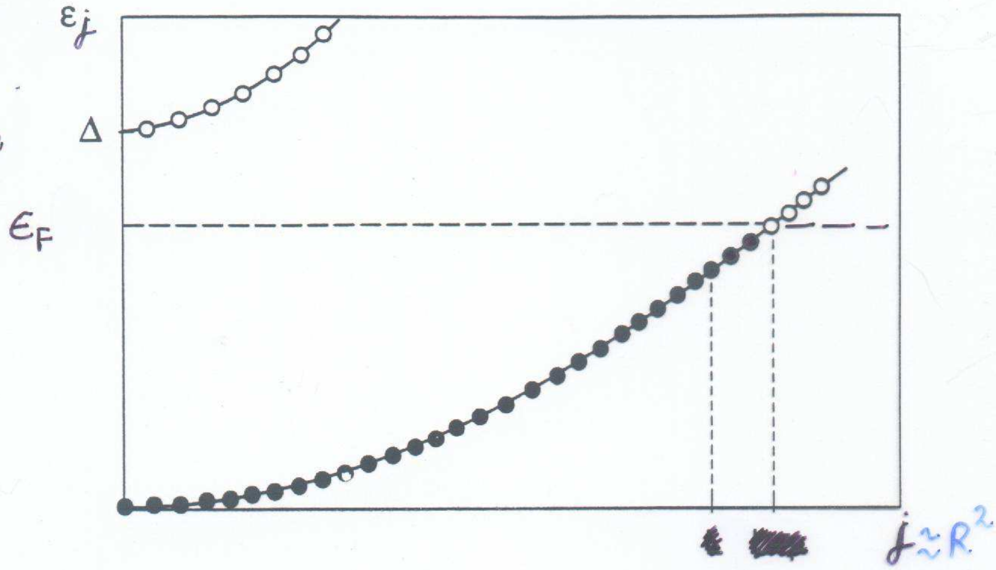
- vertex operators $:e^{in\varphi}:$ anyons at the edge

$$\langle :e^{i\varphi(z_1)}::e^{i\varphi(z_2)}: \rangle = (z_1 - z_2)^{1/3}, \qquad |z_i| = 1$$

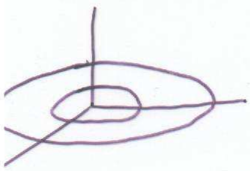
fractional statistics

The $\nu=1$ quantum incompressible fluid is like a Fermi sea in coordinate space

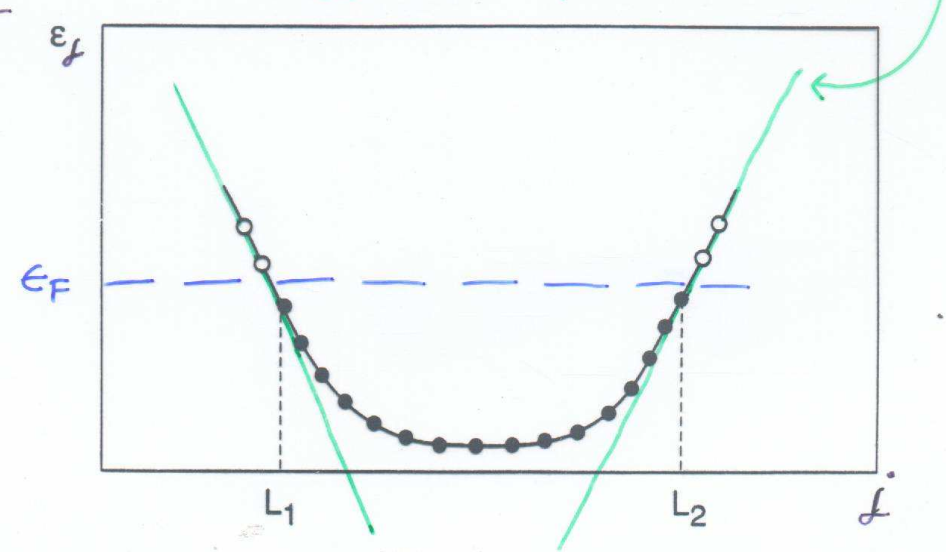
Disk

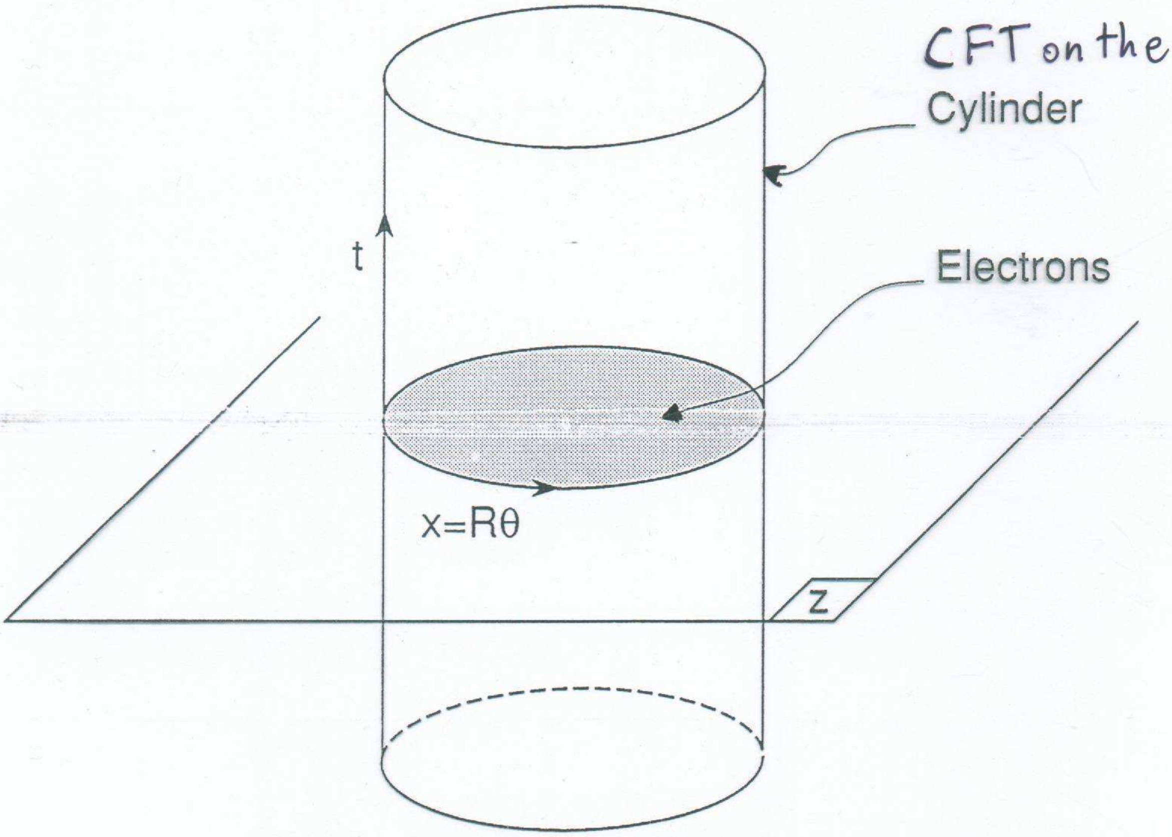


Annulus



linear approx. at the boundary
$\epsilon_j - \epsilon_F \sim v_F (j - j_F) \propto K_{boundary}$
relativistic spectrum




CFT on the
Cylinder
t
Electrons
x=Rθ
z

Edge field theory for $\nu = 1$

- Expand near the Fermi surface $|z| \cong \sqrt{N} \equiv R$

  $N \to \infty$ $\qquad \hat{\Psi}(|z| \simeq \sqrt{N}, \theta)$ up to $O(1/R)$

- relativistic spectrum $\quad \epsilon(k) = v k \quad \left(\epsilon_n = \frac{v}{R} 2\pi n \right)$

  $\to$ scale invariance $\to$ conformal invariance
  $\to$ conformal field theory on the cylinder (CFT)

- Fermi sea $\longrightarrow$ Dirac sea for the charged chiral fermion, the Weyl fermion (M. Stone)

- CFT describes universal properties of the excitations, i.e. (fractional) charge, spin and statistics, as well as dynamics to $O(1/R)$ i.e. finite-size effects

- CFT methods allow the complete exact solution of the (1+1)-dimensional effective theory:
  - extends to interacting theories for $\nu = 1/3, 2/5, \ldots$
  - proves the exactness of fractional charge
  - explains conduction experiments in strongly interacting regime

Weyl fermion Conformal Field Theory

- Field operator

$$\hat{\Psi}(Re^{i\theta}, t) \sim \frac{\eta^{1/2}}{\sqrt{2\pi R}} F(\eta) \qquad N = R^2 \to \infty$$

1^{st} Landau level   Weyl Field

$$F(\eta) = \sum_{n=-\infty}^{\infty} a_n \eta^{-n} \qquad \eta = e^{\frac{v}{R}\tau + i\theta}$$

cylinder

F is chiral $\frac{\partial}{\partial \bar{\eta}} F(\eta) = 0$ and charged $F^+ \neq F$,

$\{a_n, a_m^+\} = \delta_{n,m}$, and periodic $F(\eta e^{i2\pi}) = F(\eta)$

- Fourier modes of the charge density at the boundary

$$\rho_n = \int d\theta \, F^+ F \, e^{-in\theta}$$

- $\widehat{U(1)}$ current algebra in (1+1) dimensions

$$[\rho_n, \rho_m] = n \, \delta_{n+m,0} \qquad \text{chiral anomaly}$$

This is characteristic of a conformal field theory with Virasoro central charge $c=1$

$$L_n = \frac{1}{2} \sum_{\ell} : \rho_{n-\ell} \rho_\ell : \qquad \text{Virasoro generators}$$

$$[L_n, L_m] = (n-m) L_{n+m} + \frac{c}{12} n(n^2-1) \delta_{n+m,0}$$

- quantum numbers $\nu = 1$:

ρ_0 : $Q = n$ boundary charge = − anyon charge

L_0 : $h = \frac{n^2}{2}$ boundary spin = $\frac{1}{2}$ exchange statistics

- CFT description based on ρ_n amonts to the bosonization of the Weyl Fermion

➡ more general CFT's with $\widehat{U(1)}$ algebra

"chiral boson" ≈ "chiral Luttinger liquid"
≈ interacting fermion

Results

- CFT can be exactly solved;
- one-parameter family of theories;
- spectrum:

$$Q = \frac{n}{2k+1}, \quad h = \frac{n^2}{2k+1} \qquad n \in \mathbb{Z}, \quad k = 0, 1, 2,$$

➡ Fractional charges & statistics

- Filling fractions $\nu = \frac{1}{2k+1}$ Laughlin's states

- Vertex operators $:e^{in\varphi}:$ anyon fields at the boundary

$$\langle e^{i\varphi(\theta_1)} e^{i\varphi(\theta_2)} \rangle = \left(e^{i\theta_1} - e^{i\theta_2}\right)^{\frac{1}{2k+1}}$$

(Fubini, 1991)

Remarks

- The edge effective conformal theory of the Laughlin fluids is the chiral abelian theory $\widehat{U(1)}$ (chiral Luttinger liquid) as anticipated

- CFT construction "re-derives" the results of Laughlin theory upon using:

  - the incompressible fluid picture;
  - simple, universal assumption of $\widehat{U(1)}$ symmetry;

  $\Rightarrow$ Laughlin's theory is "universal"

- other fractions, e.g. $\nu = 2/5$: need more involved CFT

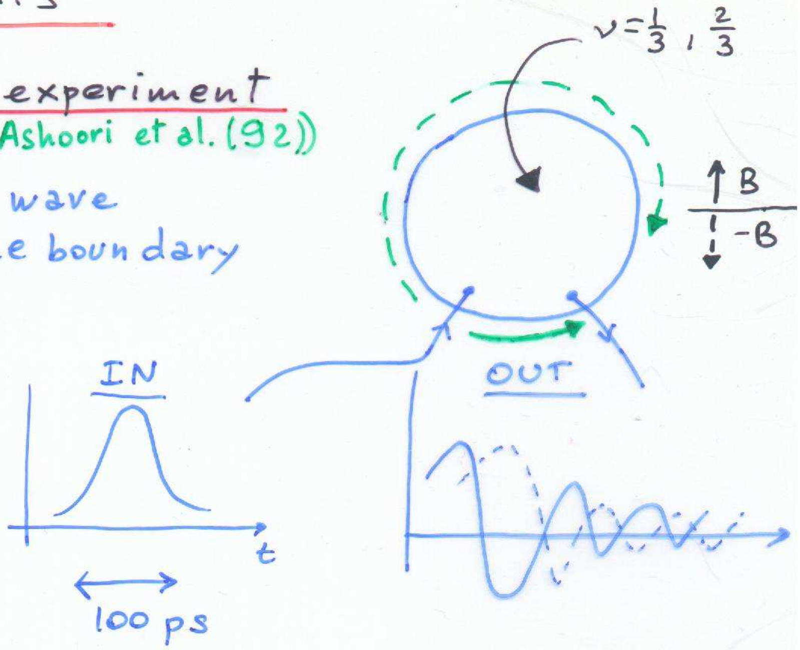
- Hall current $\leftrightarrow$ (1+1)-dimensional chiral anomaly
  $\rightarrow$ it is a topological invariant
  $\rightarrow$ it does not renormalize
  } a simple way to understand the exactness of $\sigma_{xy} = \frac{e^2}{h}\nu$

Experiments

- **Time-domain experiment** (Ashoori et al. (92))

Single chiral wave propagate at the boundary

use fast electronics ⇨

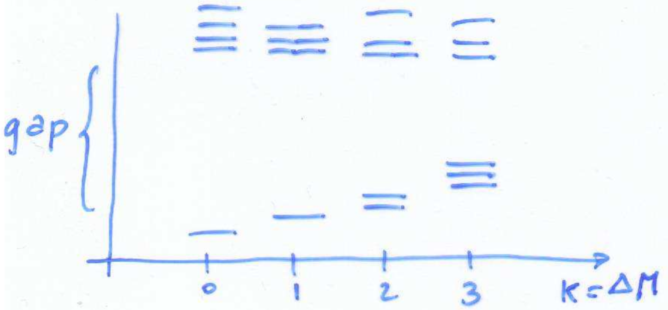


- **Numerical spectrum on the disk** (X.G. Wen, (199

\# neutral edge excitations = # descendant fields in the CFT $d(k)$

Character of $\widehat{U(1)}$ representation

$$ch_Q(\widehat{U(1)}) = \mathrm{Tr}\Big|_{\substack{\widehat{U(1)} \\ \text{rep.} \\ Q}} \left(q^{L_0}\right) = \frac{q^{L_0}}{\prod_{k=1}^{\infty}(1-q^k)} = q^{L_0}\sum_{k=0}^{\infty} q^k\, d(k)$$



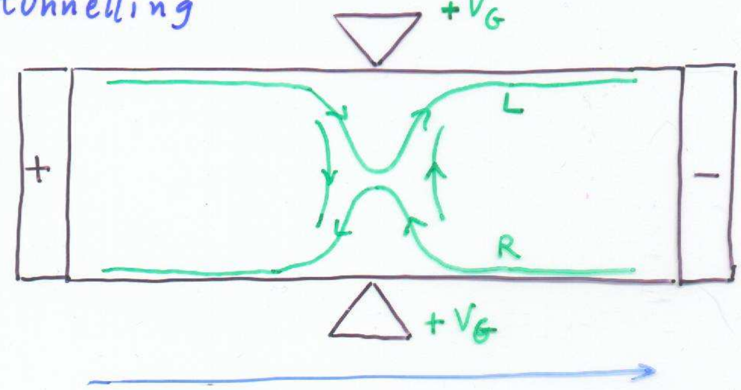
k	0	1	2	3	4	5
d	1	1	2	3	5	7

Also verified from the correlator $\dfrac{1}{(\theta - vt)^{2L_0}}$ (MacDonald et al. (94)

• Resonant tunneling through a point contact

(Kane, M. Fisher, ...)
Milliken et al. 1995

The electron fluid is squeezed at one point
Chiral & anti-chiral excitations interact
quasi-particle tunnelling



$I = G\,\Delta V$ conductance

$$S = S_{\substack{BOSON\\LEFT}} + S_{\substack{BOSON\\RIGHT}} + S_{INT}$$

$$S_{INT} = \sum_n g_n \int dx\,dt\,\delta(x)\left(e^{in\varphi_L} e^{-in\varphi_R} + h.c.\right)$$

$$\nu = \frac{1}{3} \qquad L_0 = \frac{n^2}{6}, \quad \dim g_n = 1 - \frac{n^2}{3}, \quad g_1 \sim m^{2/3}$$

only relevant interaction

$$G = \frac{e^2}{h}\,\frac{1}{3}\,\tilde{G}\left(\frac{g_1}{T^{2/3}}\right)$$

universal scaling function
$g_1 \sim$ gate voltage V_G

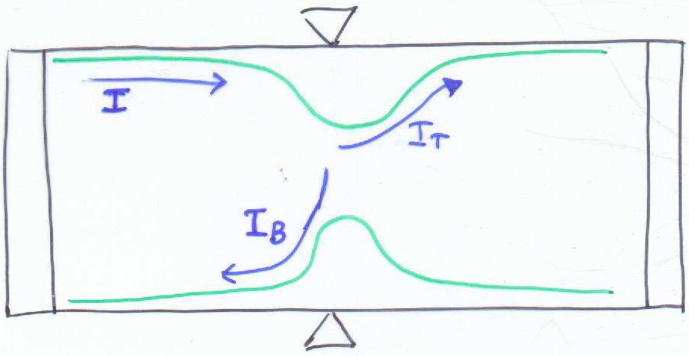
$\tilde{G}$ computed by Thermodynamic Bethe Ansatz
(Fendley, Ludwig, Saleur, 1995)

Effective field theory of edge excitations describes the conduction experiments

Shot-noise : direct measure of $Q = e/3$

$\nu = \frac{1}{3}$

(Glattli et al., 1997)
(De Picciotto et al., 1997)



Idea : fluctuations of the current are more universal than the value of the current itself : study the noise of the current

Thermal noise (equilibrium) = Johnson-Nyquist noise
needs CFT dynamics (Bethe ansatz as before)

$T=0$ shot noise (out of equilibrium) = quantum noise due to discrete nature of carriers
mostly kinematics of excitations

low current → uncorrelated tunnelling events
→ Poisson statistics

$$S_I = \langle |\delta I(\omega)|^2 \rangle_{\omega \to 0} \simeq e\, I \qquad \text{strong constriction}$$

$$\simeq \frac{e}{3}\, I_B \qquad \text{weak constriction}$$

Result derived by Kane & Fisher from chiral boson CFT; although quasi-particles are not really free d.o.f., this intuitive result holds in the limit of weak back scattering (transmission probability $|t|^2 \to 1$)
(weak interaction $g_1 \ll 1$, $I_B \ll I$)

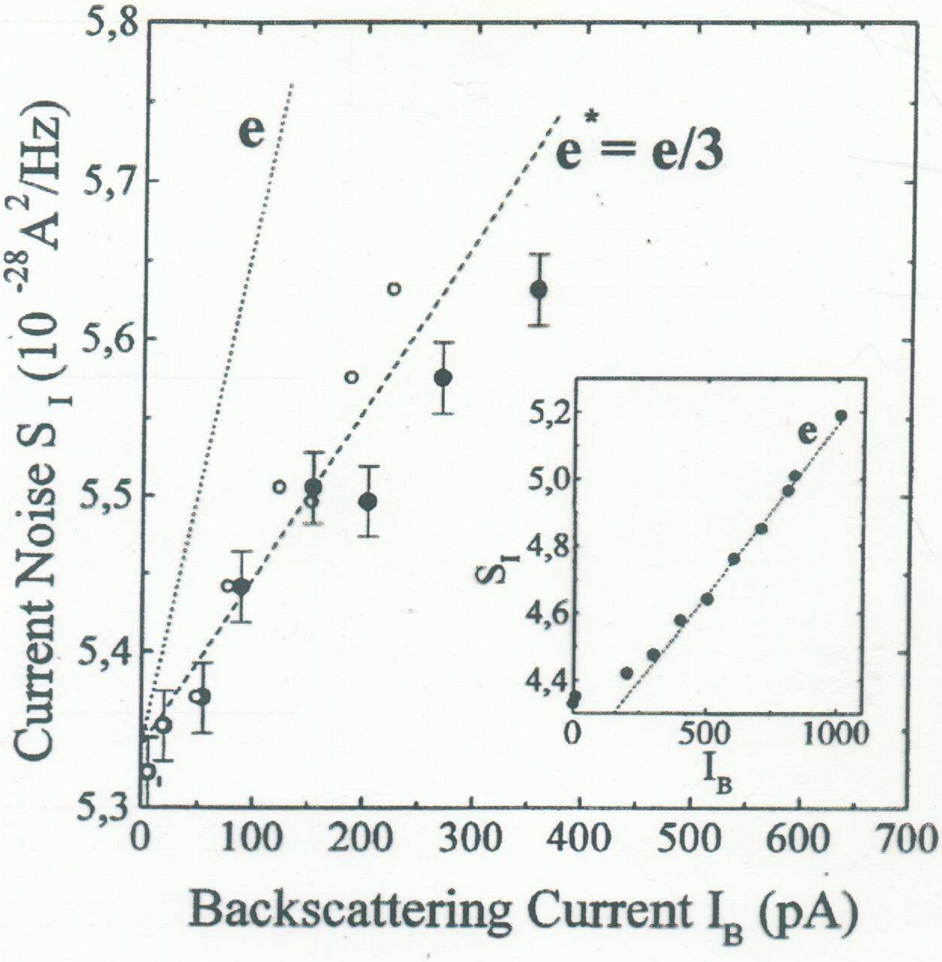


FIG. 2. Tunneling noise at $\nu = 1/3$ ($\nu_L = 2/3$) when following path A and plotted versus $I_B = (e^2/3h)V_{ds} - I$ (filled circles) and $I_B(1 - R)$ (open circles). The slopes for $e/3$ quasiparticles (dashed line) and electrons (dotted line) are shown. $\Theta = 25$ mK. Inset: data in same units showing electron tunneling for similar $G = 0.32e^2/h$ but in the IQHE regime ($\nu_L = 4$). The expected slope for electrons $2eI_B(1 - R)$ [$R = 0.68$, $I_B = (e^2/h)V_{ds} - I$] is shown. $\Theta = 42$ mK.

[Gattli et al. PRL ('97)]

Recent activity

• CFT phenomenology

Lively zoo of theories can be used to describe:

- other plateaux (Jain's states, $\nu = \frac{2}{5}, \frac{2}{7}, \dots$)
- special geometries (multilayers, interfaces, ..)
- conduction experiments (transport theory, use of integrable field theories)

too many CFT's !! need criteria:

➡ incompressible fluid $\leftrightarrow$ W_∞ symmetry

➡ use CFT's with W_∞ symmetry or suitable breakings of this (A.C., Trugenberger, Zemba, 92 – 01)

• (Non-commutative) Chern-Simons theory

• (2+1)-dim C–S theory $\approx$ (1+1)-dim CFT

U(1) gauge symmetry $\qquad$ $\widehat{U(1)}$ current algebra

$$S_{C\text{-}S} = \frac{k}{4\pi} \int d^3x \; A\, d A \qquad \nu = \frac{1}{k} = 1, \frac{1}{3}, \frac{1}{5}, \dots$$

• non-commutative version (Susskind, 01)

$$S_{C\text{-}S} = \frac{k}{4\pi} \int \hat{A} * d\hat{A} + \frac{2}{3} \hat{A} * \hat{A} * \hat{A}, \qquad f * g(x) = e^{i\theta \partial_1 \partial_2} f(x_1) g(x_2)\Big|_x$$

$$\theta = \frac{1}{2\pi\rho_0}$$

it looks like being fancy:
open strings, holography, ...

Interest of Fractional QHE

- Experimentally : physics of next generation of semi-conductors

- Theoretically : i) universality

  ii) high precision

  iii) new state of matter with free "quarks"

Remarks

(i) + (ii) suggest that:

"Kinematics" dominates "dynamics"

↑

algebraic conditions due to symmetries

(iii) may suggest non-perturbative phenomena in other contexts, as superfluidity has suggested SSB in the Standard Model of particles

Phylosophy : fundamental physics, but not elementary-particle physics