Condensed Matter applications of 2D conformal field theories

A. Cappelli (INFN, Florence)

Success stories

- quantum spin chains (Haldane, ...)
- (multichannel) Kondo effect (Affleck, A.W.W. Ludwig, ...)
- quantum Hall effect (X.G. Wen, Read, M. Fisher, Kane, Saleur, ...)

cf. - Amsterdam Workshop "Flux, Charge, Topology, Statistics" July 2003 & 2001 (K. Schoutens)

- EC Network "EUCLID: Integrable models and applications" (E. Corrigan)
- A. Cappelli, G. Mussardo eds., "Statistical Field Theories" NATO conf. Como 2001, Kluwer (2002)

2D Conformal field theories: Intro

Consistent formalism developed for String Theory (Belavin, Polyakov, Zamolodchikov; Cardy; ....) based on the math of ∞-dim Lie algebras

$$z = x + iv\tau \qquad \bar{z} = x - iv\tau$$

$z = f(w)$ analytic

$$ds^2 = dx^2 + d\tau^2 = dz\,d\bar{z} = \left|\frac{df}{dz}\right|^2 dw\,d\bar{w}$$ conformal

implied by scale and translation invariances
$z = \lambda w$ $\qquad$ $z = w + c$

massless correlators are covariant

$$\langle \phi_\Delta(z,\bar{z})\,\phi_\Delta(0,0)\rangle = \frac{1}{|z|^{2\Delta}} = \frac{\lambda^{-2\Delta}}{|w|^{2\Delta}} \qquad z = \lambda w$$

infinitesimal transformations $\quad z = w + \epsilon(w)$

$$\phi(w + \epsilon(w)) = \left(1 + \epsilon(w)\frac{\partial}{\partial w}\right)\phi(w)$$

$$\epsilon(w) = \sum_n \epsilon_n w^{n+1} \longrightarrow L_n = -z^{n+1}\frac{\partial}{\partial z}$$

Virasoro algebra

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\,\delta_{n+m,0}$$

c = central charge, characteristic of each conformal theory

Hamiltonian density is part of stress tensor

$$\mathcal{H} = T_{00}(z,\bar{z})$$

analytic, i.e. chiral, splitting

$$\begin{cases} \partial_\mu T_{\mu\nu} = 0 \\ T_\mu{}^\mu = 0 \end{cases} \longrightarrow \begin{cases} \frac{\partial}{\partial \bar{z}} T_{zz} = 0 \\ T_{z\bar{z}} = 0 \end{cases} \quad T_{zz} \text{ analytic}$$

$$T_{zz}(z) \quad , \quad T_{\bar{z}\bar{z}}(\bar{z})$$

the same for fields, that carry a representation of the Virasoro algebra of dim h (and $\bar{h}$)

$$\phi_\Delta(z,\bar{z}) = \phi_h(z)\, \bar{\phi}_{\bar{h}}(\bar{z}) \qquad \Delta = h + \bar{h}$$

correlators factorize into chiral $\times$ antichiral blocks

CFT main data: $\begin{cases} c & \text{central charge} \\ \{\Delta_n\} & \text{set of scale dimensions} \end{cases}$

spectra determined by representation theory and some physical conditions

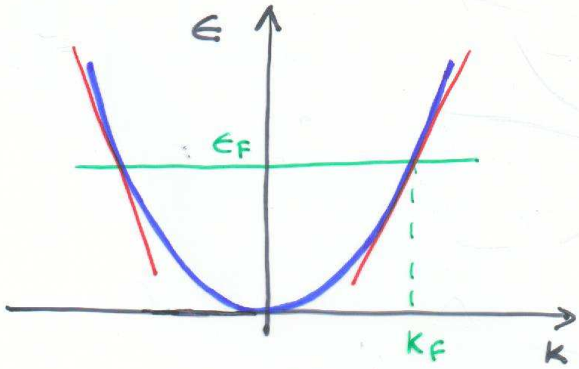
<u>BIG ZOO</u> of massless interacting theories

+ massive integrable "deformations"

- exact S-matrix bootstrap
- form factors expansion
- thermodynamic <u>Bethe ansatz</u>

Luttinger theory ≈ c=1 scalar field CFT

Interacting (spinless) fermion in one dim:



- expand around $k \sim \pm k_F$
  → massless "relativistic"
- bosonize the current

$$J_\mu = (J_z, J_{\bar z}) \quad , \quad \frac{\partial}{\partial \bar z} J_z = 0 \quad , \quad J_z = \psi_R^+ \psi_R$$

$$J_z = i \frac{\partial}{\partial z} \varphi \quad , \quad J_{\bar z} = -i \frac{\partial}{\partial \bar z} \varphi$$

φ free scalar field, compactified $\varphi \approx \varphi + 2\pi r$

stress tensor $T_{zz} = \frac{1}{2} : J_z J_z :$ (Sugawara)

$$H = \int dx \, \mathcal{H} = \int dx \left(T_{zz} + T_{\bar z \bar z} \right) = \frac{1}{2} \int dx : (\partial_\tau \varphi)^2 + (\partial_x \varphi)^2 :$$

(Tomonaga - Luttinger)

correlators $\langle : e^{i\beta\varphi(z)} : : e^{-i\beta\varphi(w)} : \rangle = |z-w|^{-2\beta^2}$ $\Delta = \beta^2$

radius $\sqrt{2} \leq r < \infty$ related to marginal coupling g
$c=1$ all along the line

$$\left\{ \Delta_{m,n} = h_{m,n} + \bar h_{m,n} \;\middle|\; h = \frac{1}{2}\left(\frac{m}{2r} + nr\right)^2, \bar h = \frac{1}{2}\left(\frac{m}{2r} - nr\right)^2, m,n \in \mathbb{Z} \right\}$$

$\widehat{U(1)}$ current algebra: $J_z = \sum_n \rho_n z^{-n-1}$

$$[\rho_n, \rho_m] = n \, \delta_{n+m,0} \qquad \rho_0 = \beta \text{ charge}$$

elementary excitations have fractional charge

$\widehat{SU(2)}_k$ symmetry: Wess–Zumino–Witten theory

consider now spinful fermion $\psi_{R\pm}$, $\psi_{L\pm}$ $(\sigma = \pm)$
two currents:

$$J_z = \psi^+_{R+}\psi_{R+} + \psi^+_{R-}\psi_{R-} = i\frac{\partial}{\partial z}\varphi_c \qquad \text{charge}$$

$$J^3_z = \psi^+_{R+}\psi_{R+} - \psi^+_{R-}\psi_{R-} = i\frac{\partial}{\partial z}\varphi_s \qquad \text{spin}$$

they bosonize independently and decouple in $\mathcal{H}$
$\longrightarrow$ spin – charge separation

$$J^+_z = \psi^+_{R+}\psi_{R-}, \quad J^-_z = \psi^+_{R-}\psi_{R+}$$

$$J^\pm_z = e^{\pm i\sqrt{2}\varphi_s} \qquad \text{at radius } r_s = \sqrt{2} \text{ free fermion}$$

$J^3, J^\pm$ generate $\widehat{SU(2)}_1$ current algebra $c=1$

spin coupling is not marginal, r_s is fixed

Another theory of bosonic currents, the WZW model, realizes other, discrete points with SU(2) symmetry

$$\widehat{SU(2)}_k, \quad c = \frac{3k}{k+2}, \quad k = 1, 2, \ldots$$

Δ_n spectra and exact solution thanks to the extended symmetry

$$T_{zz} \propto \sum_{a=1,2,3} :J^a_z J^a_z: \qquad \text{Sugawara form}$$

Landau levels: one-electron states

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} - e\vec{A}\right)^2 , \quad A = \frac{B}{2}(-y, x) , \quad z = x + iy, \quad \partial = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)$$

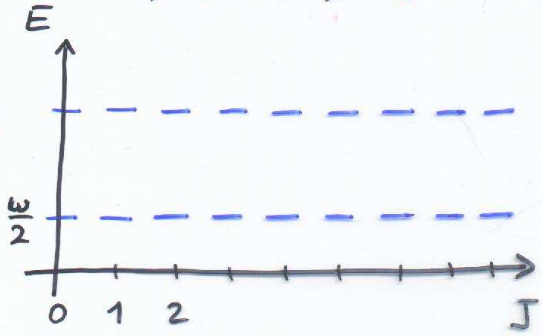
$$\begin{cases} c = \dfrac{z}{2\ell} + \ell\bar{\partial} \\ c^{+} = \dfrac{\bar{z}}{2\ell} - \ell\partial \end{cases} \qquad \begin{cases} b = \dfrac{\bar{z}}{2\ell} + \ell\partial \\ b^{+} = \dfrac{z}{2\ell} - \ell\partial \end{cases}$$

$$[c, c^{+}] = 1 , \quad [b, b^{+}] = 1$$

$$[c, b] = [c, b^{+}] = 0$$

$$\mathcal{H} = \hbar\omega\left(c^{+}c + \frac{1}{2}\right) \qquad \omega = \frac{eB}{mc}$$

$$J = \vec{x}\wedge\vec{p} = b^{+}b - c^{+}c$$



magnetic length $\ell = \sqrt{\dfrac{2\hbar c}{eB}}$

$\ell = 100 \div 1000\ \text{Å}$

minimal orbit $\approx$ unit flux

$$\pi\ell^2 B = \phi_0 = \frac{hc}{e}$$

Lowest Landau level:

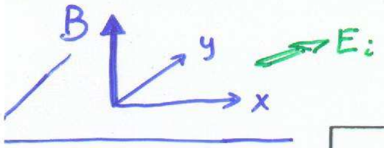
$$0 = c\,\varphi_{0,j}(z,\bar{z}) = \left(\ell\bar{\partial} + \frac{z}{2\ell}\right)\varphi_{0,j} , \qquad \varphi_{0,j} = e^{-\frac{1}{2}\frac{|z|^2}{\ell^2}}\left(\frac{z}{\ell}\right)^j \frac{1}{\sqrt{j!}}$$

$J\varphi_{0,j} = j\,\varphi_{0,j}$ orbitals peaked at $|z|^2 = \ell j$

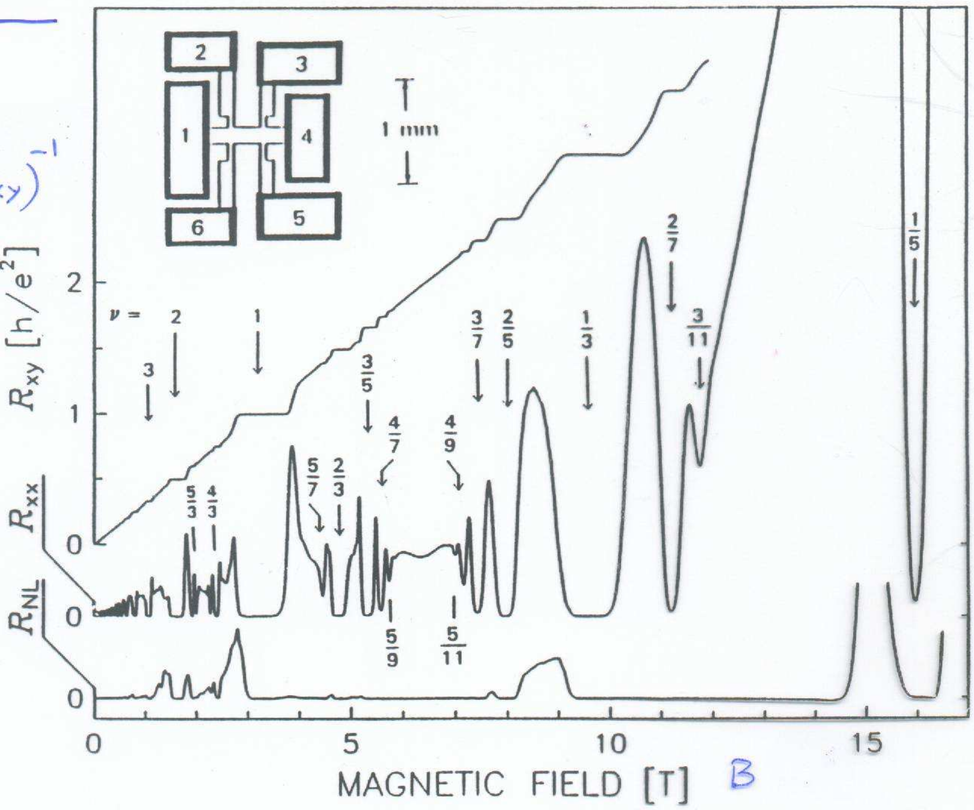
- degeneracy D_A = # of states in a given area A

$$D_A = \frac{BA}{\phi_0} = \frac{\phi}{\phi_0} = \#\ \text{fluxes}$$

- filling fraction $\nu = \dfrac{N_{\text{electrons}}}{D_A} = \dfrac{N}{BA/\phi_0} = \underbrace{\dfrac{\rho_0}{B}}_{}\dfrac{hc}{e}$ uniform



$R_{xy} = (\sigma_{xy})^{-1}$



$$V_i = R_{ij} I_j \quad , \quad i,j = x,y \qquad \text{or} \qquad J_i = \sigma_{ij} \mathcal{E}_j$$

$$R_{ij} = \begin{pmatrix} R_{xx} & R_{xy} \\ R_{xy} & R_{xx} \end{pmatrix} = \begin{pmatrix} 0 & R_{xy} \\ R_{xy} & 0 \end{pmatrix} \quad \text{at centers of plateaux}$$

$$\sigma_{ij} \propto R_{ij}^{-1} = \begin{pmatrix} 0 & \sigma_{xy} \propto R_{xy}^{-1} \\ \sigma_{xy} & 0 \end{pmatrix}$$

Experimental results:

- $\sigma_{xx} = 0$ No Ohmic conduction $\rightarrow$ Gap

- $\sigma_{xy} = \frac{e^2}{h} \nu$, $\nu = 1 (\pm 10^{-8}), 2, 3, \dots, \frac{1}{3} (\pm 10^{-6}), \dots$

  high precision: very stable ground state with uniform density

  $$\rho(x,y) \cong \rho_0 = \frac{eB}{hc} \nu$$

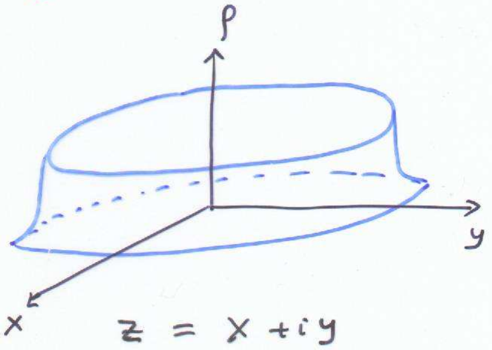
- universality

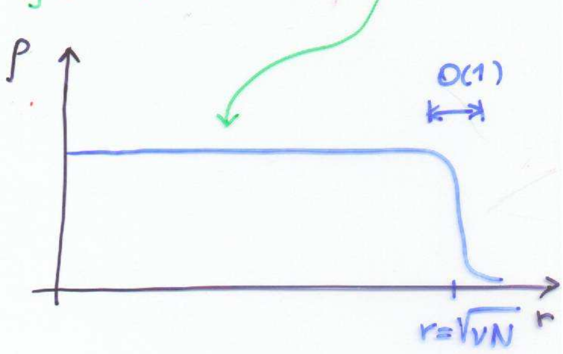
- non-trivial pattern of fractional values of ν: fractional Hall effect

The Laughlin incompressible fluid

Electrons behave as a droplet of liquid without sound waves

$$\begin{cases} \text{Incompressible} \equiv \text{density waves have a gap} \\ \text{Fluid} \equiv \rho(\vec{x}) = \text{const.} \end{cases}$$





$z = x + iy$

A = area of the droplet

N = # of electrons

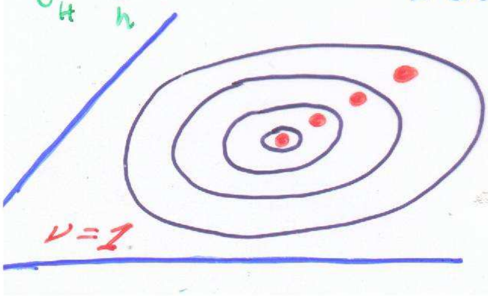
$\mathcal{D}_A = \dfrac{BA}{2\pi \frac{\hbar c}{e}}$ = # of degenerate Landau orbitals

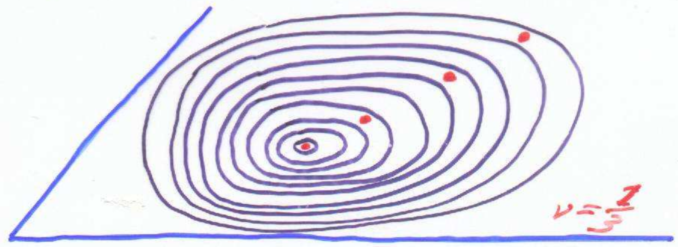
→ $\rho = \dfrac{N}{A}$ = electron density

→ $\nu = \dfrac{N}{\mathcal{D}_A} \propto \dfrac{N}{BA}$ = filling fraction $= 1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots$

= density for quantum-mechanical problem

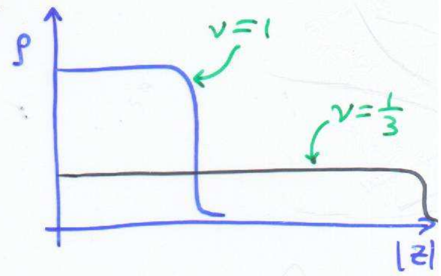
$\sigma_H = \dfrac{e^2}{h}\nu$





- Laughlin's wave function $\nu = \frac{1}{m} = 1, \frac{1}{3}, \frac{1}{5}, \dots$

$$\Psi_{g.s.}(z_1, \dots, z_N) = \prod_{1=i<j}^{N} (z_i - z_j)^m \, e^{-\frac{1}{2}\sum \frac{|z_i|^2}{\ell^2}}$$

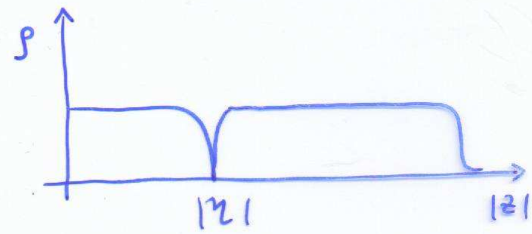


- $\nu = 1$ obvious
  Filled Landau level
  gap $= \omega_c = \frac{eB}{mc}$

- $\nu = 1/3, 1/5, \dots$ highly non-trivial
  due to repulsive electron-electron interaction
  gap $\sim O(e^2/\ell)$ $\quad\ell =$ magnetic length $\propto 1/\sqrt{B}$

- quasi-hole excitation $\simeq$ vortex

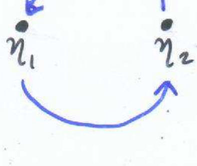
$$\Psi(\eta; z_1 \dots z_N) = \prod_{i=1}^{N} (\eta - z_i) \prod_{i<j} (z_i - z_j)^m \, e^{-\frac{1}{2}\sum |z_i|^2/\ell^2}$$



- For $m > 1$, it has fractional charge $Q = \frac{e}{m}$

- For $m > 1$, it has fractional statistics $\frac{\theta}{\pi} = \frac{1}{m}$

$$\Psi(\eta_1, \eta_2; z_1 \dots z_N) \sim (\eta_1 - \eta_2)^{\frac{1}{m}}$$



$$\Psi(\eta_1 - \eta_2 \to e^{i\pi}(\eta_1 - \eta_2)) = e^{\frac{i\pi}{m}} \Psi(\eta_1, \eta_2$$

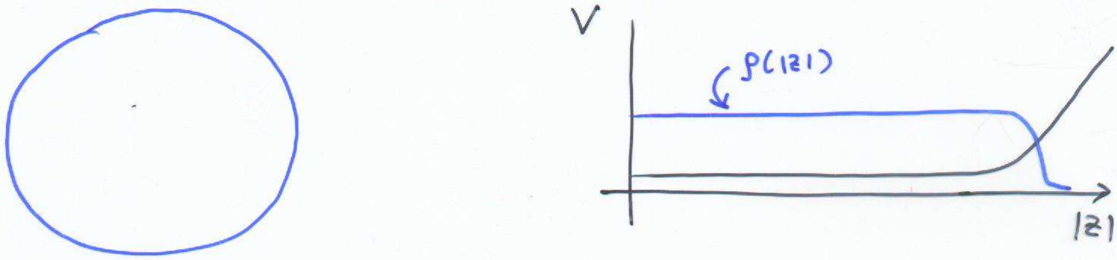
Fractiona statistics $\frac{\theta}{\pi} = \frac{1}{m} = \frac{1}{3}, \frac{1}{5}, \ldots$

quasi-hole is an anyon (Wilczek et al)

- fractional statistics is a long-range correlation of vortices, independent of distance, i.e. "topological"
  (nicely modelled by conformal field theory or Chern-Simons gauge field)

Edge excitations of the incompressible fluid

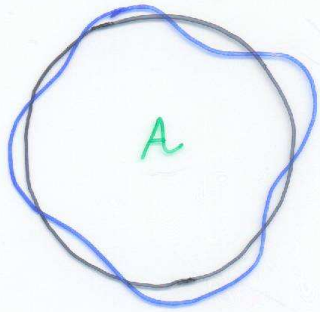
The sample has a boundary, with a confining potential; take e.g. a disk:



The incompressible fluid satisfies $\rho(|z|) = \rho_0$, i.e. (2+1)-dimensional waves have a high gap and can be neglected.
But:

- the boundary shape can fluctuate:

→ "neutral" edge excitations



- almost gapless
- excitations satisfy $A = \text{cost}$

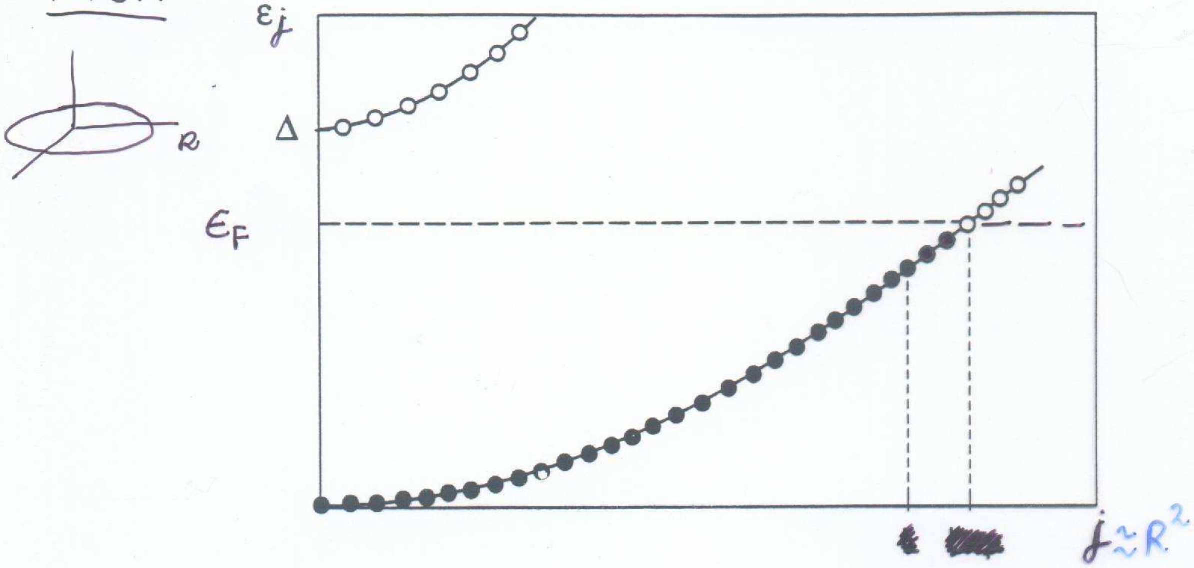
$$N = \int d^2x\, \rho(x) = \rho_0 \cdot A \quad , \quad N, \rho_0 = \text{cost}$$

→ area-preserving diffeomorphisms of the plane
= W_∞ symmetry (my work with C. Trugenberger & G. Zemba (92))

- $\nu = 1$ obvious:
the filled Landau level is like a Fermi sea
edge excitations = particle-hole excitations at the Fermi surface

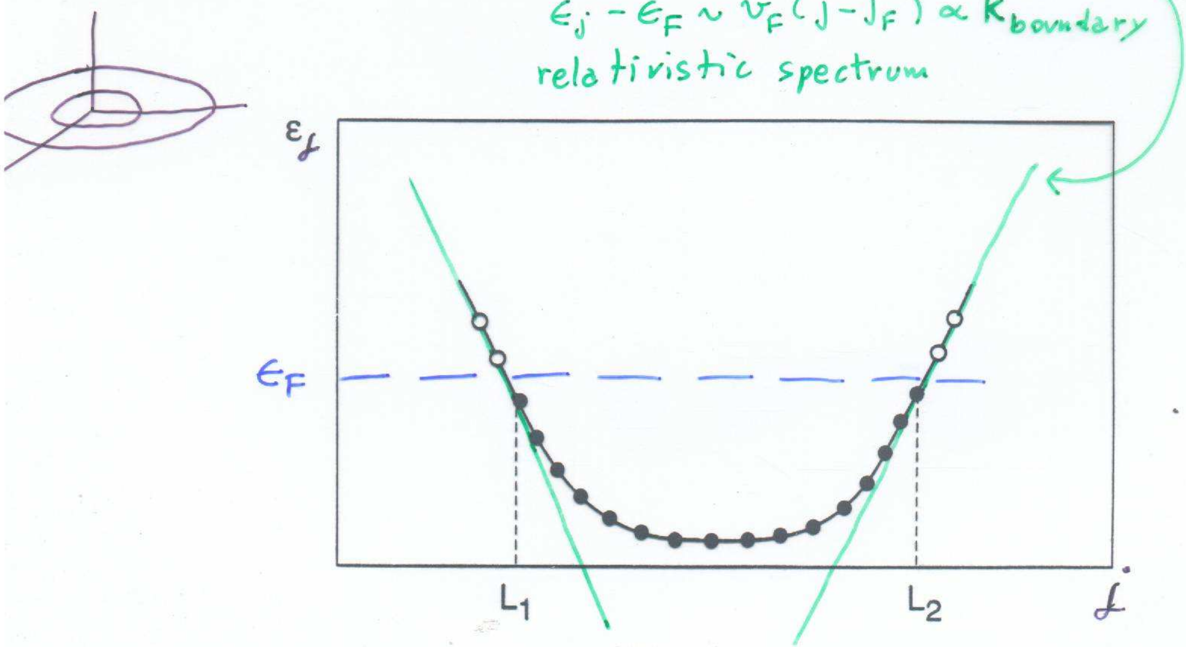
The $\nu = 1$ quantum incompressible fluid is like a Fermi sea in coordinate space

Disk

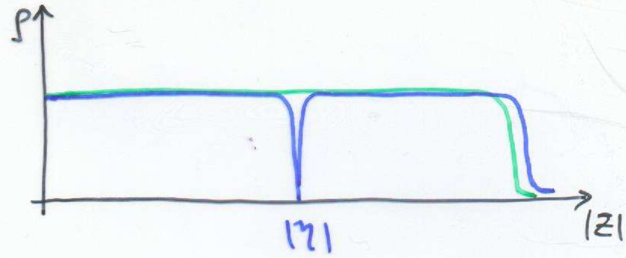


Annulus

linear approx. at the boundary
$\epsilon_j - \epsilon_F \sim v_F (j - j_F) \propto K_{boundary}$
relativistic spectrum



• charged quasi-hole excitation also observed at the edge:



depleted density is spilled at the edge
→ charged excitation at the edge

Conclusion: both excitations of the incompressible fluid can be detected at the boundary

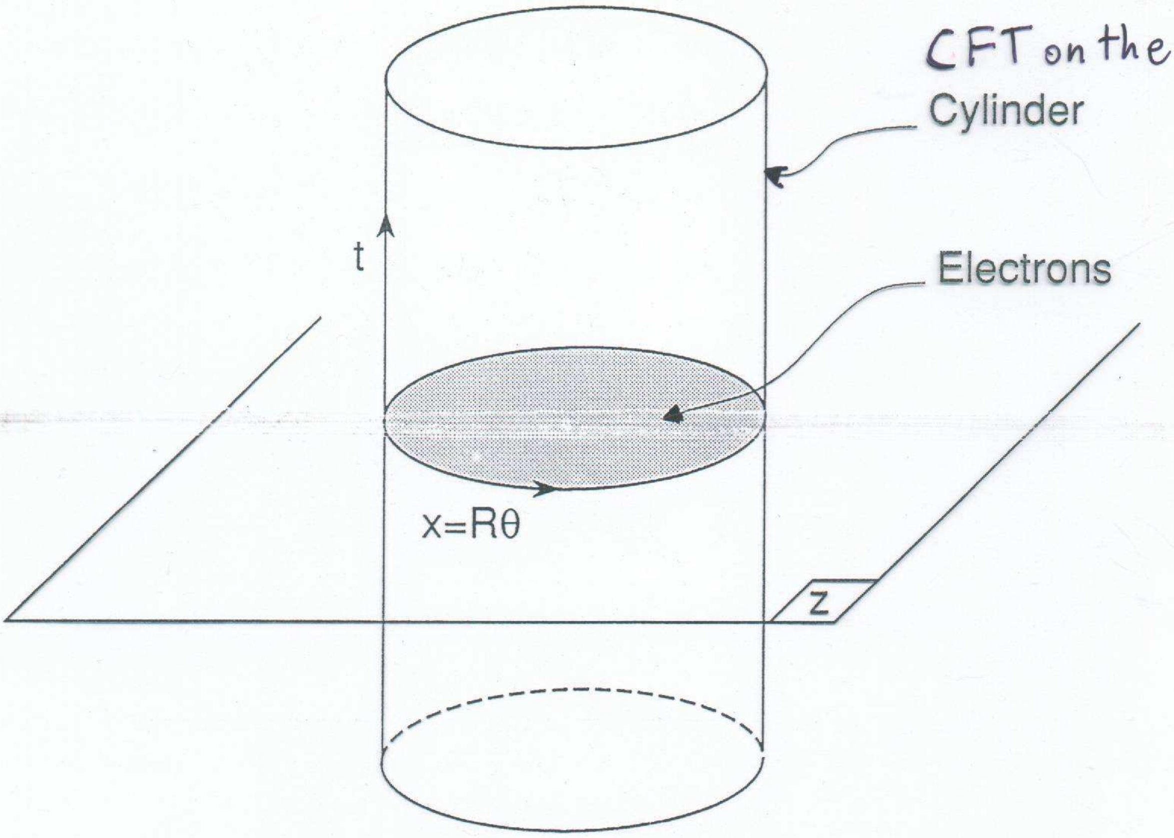
Idea: reduced field theory in (1+1) dimensions
≡ conformal field theory (X.G.Wen)

Main Features:

• effective field theory, valid for low-energy, long-range physics
(as for critical phenomena in statistical mech.)

• it is sufficient to describe conduction experiments
(μV, μA, mK)

• it can be derived for $\nu=1$; it is guessed for $\nu=\frac{1}{3},\frac{1}{5},\ldots$
universality ≡ effective theory determined by general features of the system like dimensionality, type of d.o.f., symmetries


CFT on the
Cylinder
t
Electrons
x=Rθ
z

Edge field theory for $\nu = 1$

- Expand near the Fermi surface $|z| \cong \sqrt{N} \equiv R$

  $N \to \infty \qquad \hat{\Psi}(|z| \simeq \sqrt{N}, \theta)$ up to $O(1/R)$

- relativistic spectrum $\quad \epsilon(k) = v k \quad \left(\epsilon_n = \frac{v}{R} 2\pi n \right)$

  $\rightarrow$ scale invariance $\rightarrow$ conformal invariance
  $\rightarrow$ conformal field theory on the cylinder (CFT)

- Fermi sea $\longrightarrow$ Dirac sea for the charged chiral fermion, the Weyl fermion (M. Stone)

- CFT describes universal properties of the excitations, i.e. (fractional) charge, spin and statistics, as well as dynamics to $O(1/R)$ i.e. finite-size effects

- CFT methods allow the complete exact solution of the (1+1)-dimensional effective theory:
  - extends to interacting theories for $\nu = 1/3, 2/5, \ldots$
  - proves the exactness of fractional charge
  - explains conduction experiments in strongly interacting regime

- quantum numbers $\nu = 1$:

  ρ_0 : $Q = n$ boundary charge $= -$ anyon charge

  L_0 : $h = \frac{n^2}{2}$ boundary spin $= \frac{1}{2}$ exchange statistics

- CFT description based on ρ_n amonts to the bosonization of the Weyl Fermion

→ more general CFT's with $\widehat{U(1)}$ algebra

"chiral boson" $\approx$ "chiral Luttinger liquid"
$\approx$ interacting fermion

Results

- CFT can be exactly solved;
- one-parameter family of theories;
- spectrum:

$$Q = \frac{n}{2k+1} , \quad h = \frac{n^2}{2k+1} \qquad n \in \mathbb{Z}, \quad k = 0, 1, 2,$$

→ Fractional charges & statistics

- filling fractions $\nu = \frac{1}{2k+1}$ Laughlin's states
- Vertex operators $:e^{in\varphi}:$ anyon fields at the boundary

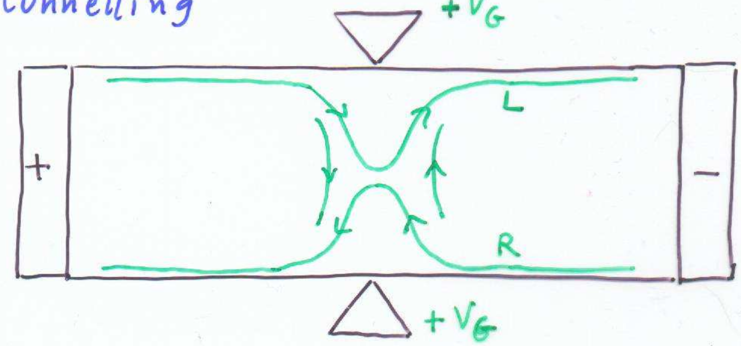
$$\langle e^{i\varphi(\theta_1)} e^{i\varphi(\theta_2)} \rangle = \left(e^{i\theta_1} - e^{i\theta_2} \right)^{\frac{1}{2k+1}}$$

(Fubini, 1991)

• Resonant tunneling through a point contact

(Kane, M. Fisher, ...)
Milliken et al. 1995

The electron fluid is squeezed at one point
Chiral & anti-chiral excitations interact
quasi-particle tunnelling



$I = G\,\Delta V$ conductance

$$S = S_{\substack{BOSON\\LEFT}} + S_{\substack{BOSON\\RIGHT}} + S_{INT}$$

$$S_{INT} = \sum_n g_n \int dx\,dt\; \delta(x)\left(e^{in\varphi_L}\, e^{-in\varphi_R} + h.c.\right)$$

$\nu = \frac{1}{3}$ $\quad L_0 = \frac{n^2}{6}\,,\quad \dim g_n = 1 - \frac{n^2}{3}\,,\quad g_1 \sim m^{2/3}$ only relevant interaction

$$G = \frac{e^2}{h}\,\frac{1}{3}\,\tilde{G}\left(\frac{g_1}{T^{2/3}}\right)$$ universal scaling function
$g_1 \sim$ gate voltage V_G

$\tilde{G}$ computed by Thermodynamic Bethe Ansatz
(Fendley, Ludwig, Saleur, 1995)

Effective field theory of edge excitations describes the conduction experiments

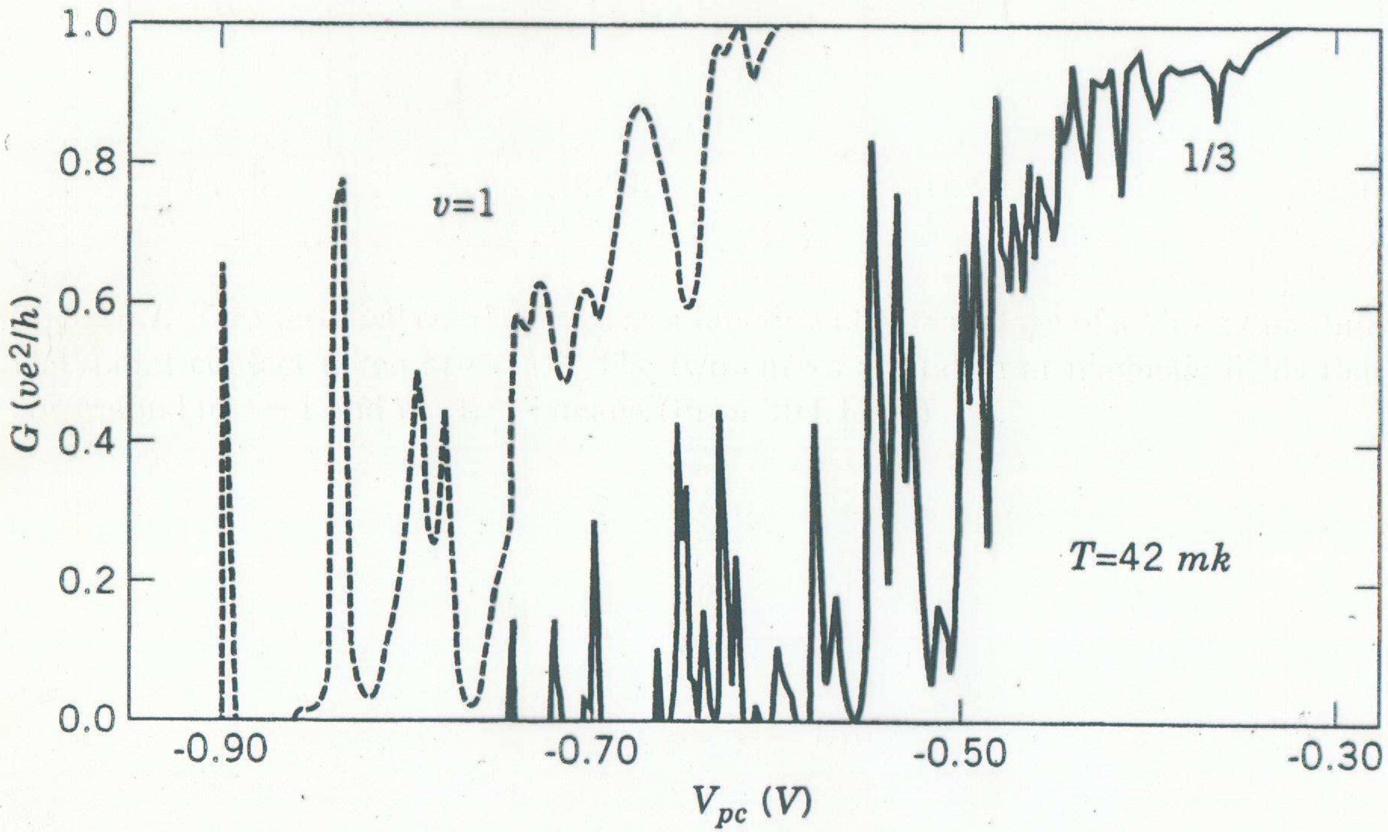
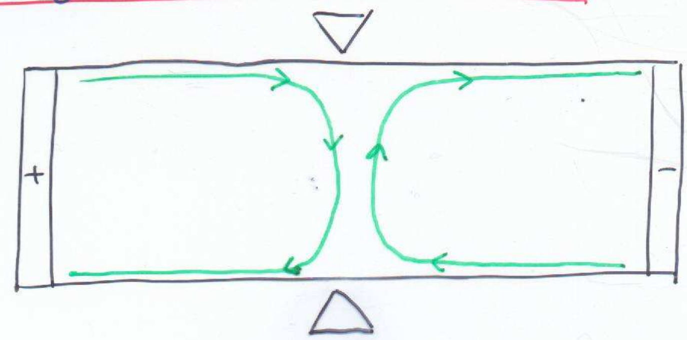


Figure 4.7. Two-terminal conductance as a function of gate voltage of a GaAs quantum Hall point contact taken at 42 mK. The two curves are taken at magnetic fields that correspond to $\nu = 1$ and $\nu = 1/3$ plateaus. (From Ref. [29].) (Milliken et al, S.S.C. 1996)

- Other limit : high $V_G \to$ electron tunnelling

two distinct fluids



Same interaction with electron field

$$Q = \frac{n}{m} = 1, \quad n = m, \quad \dim g_m = 1 - \frac{m^2}{m} = 1 - m \leq 0 \qquad \nu = \frac{1}{m}$$

irrelevant interaction

Perturbative in g_m is OK

$$G \sim g_m^2 / T^{2-2m} \sim T^{2m-2} \qquad \left(\nu = \frac{1}{m}\right)$$

$\nu = 1 \longrightarrow$ no T-dependence as $T \to 0$
$\nu = 1/3$ $\qquad T^4$-vanishing
} it is observed OK

- Exact form of $G(g_1/T^{2/3})$ (Saleur et al.)

The CFT + boundary interaction has been identified by Saleur et al. as the "boundary Sine-Gordon model". One obtains the exact shape of resonance peaks.

- Experimental peaks scale with $T^{-2/3}$ as predicted
- the shape matches almost OK $\to$ figure

$\nu = \frac{1}{3}$

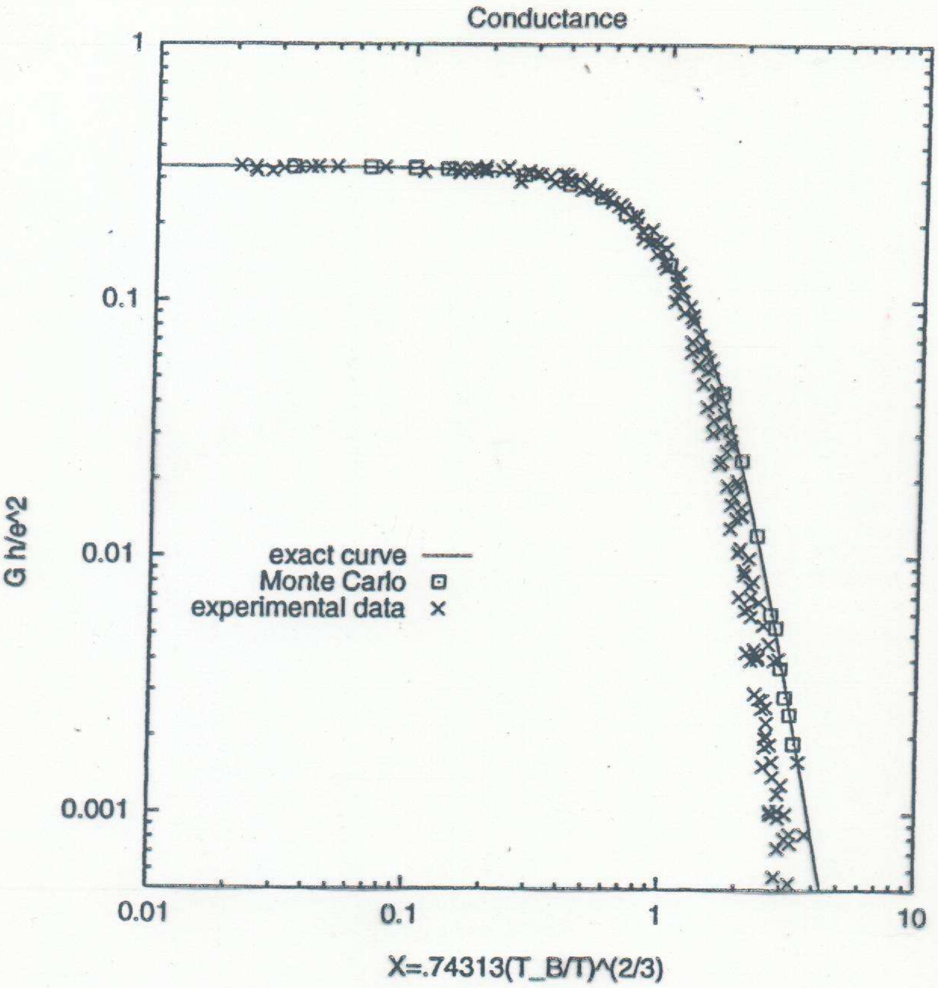


Figure 4.14. Log-log scaling plot of the lineshape of resonances at different temperatures from Ref. [29]. The x axis is rescaled by $T^{2/3}$. The crosses represent experimental data at temperatures between 40 and 140 mK. The squares are the results of the Monte Carlo simulation, and the solid line is the exact solution from Ref. [31].

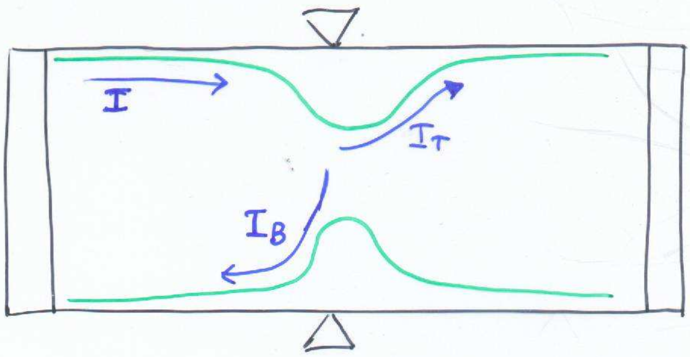
[29] Milliken et al

[31] Fendley et al

Shot-noise : direct measure of $Q = e/3$

$\nu = \frac{1}{3}$

(Glattli et al., 1997)
(De Picciotto et al., 1997)



Idea : fluctuations of the current are more universal than the value of the current itself : study the noise of the current

Thermal noise (equilibrium) = Johnson-Nyquist noise
needs CFT dynamics (Bethe ansatz as before)

$T=0$ shot noise (out of equilibrium) = quantum noise due to discrete nature of carriers
mostly kinematics of excitations

low current → uncorrelated tunnelling events
→ Poisson statistics

$$S_I = \langle |\delta I(\omega)|^2 \rangle_{\omega \to 0} \simeq e\, I \qquad \text{strong constriction}$$

$$\simeq \frac{e}{3} I_B \qquad \text{weak constriction}$$

Result derived by Kane & Fisher from chiral boson CFT; although quasi-particles are not really free d.o.f., this intuitive result holds in the limit of weak back scattering (transmission probability $|t|^2 \to 1$)
(weak interaction $g_1 \ll 1$, $I_B \ll I$)

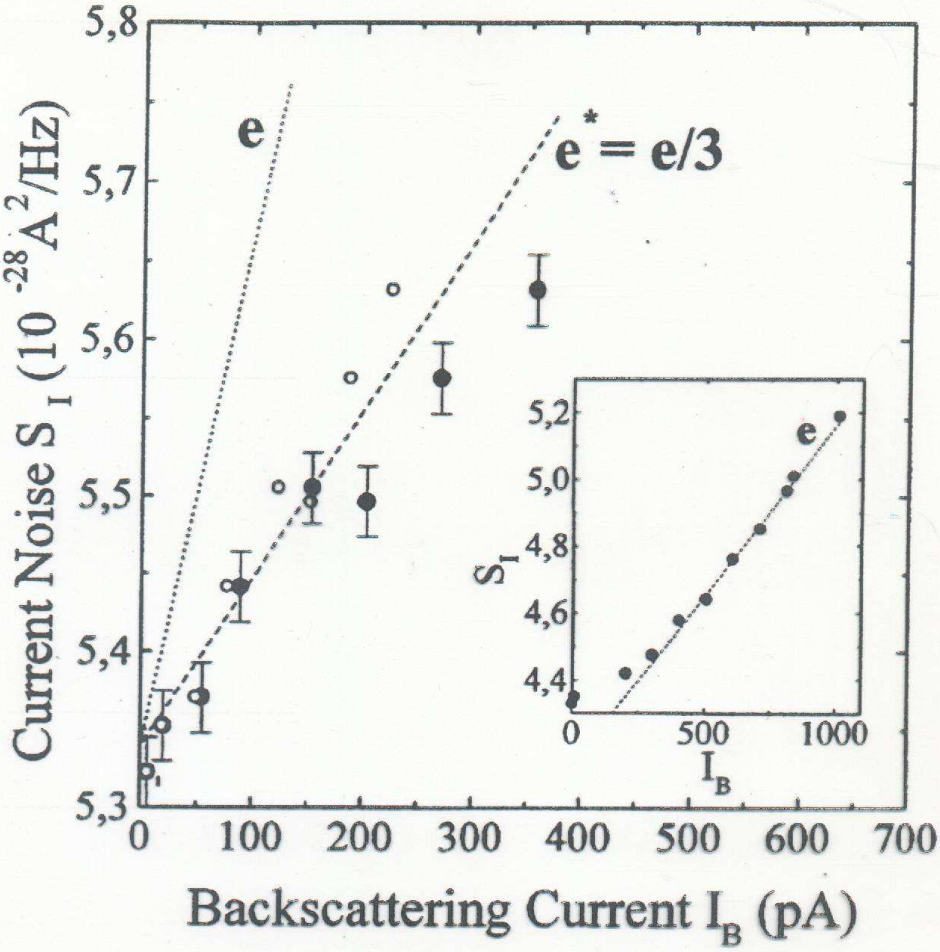


FIG. 2. Tunneling noise at $\nu = 1/3$ ($\nu_L = 2/3$) when following path A and plotted versus $I_B = (e^2/3h)V_{ds} - I$ (filled circles) and $I_B(1 - R)$ (open circles). The slopes for $e/3$ quasiparticles (dashed line) and electrons (dotted line) are shown. $\Theta = 25$ mK. Inset: data in same units showing electron tunneling for similar $G = 0.32e^2/h$ but in the IQHE regime ($\nu_L = 4$). The expected slope for electrons $2eI_B(1 - R)$ [$R = 0.68$, $I_B = (e^2/h)V_{ds} - I$] is shown. $\Theta = 42$ mK.

[Gatti et al. PRL ('97)]

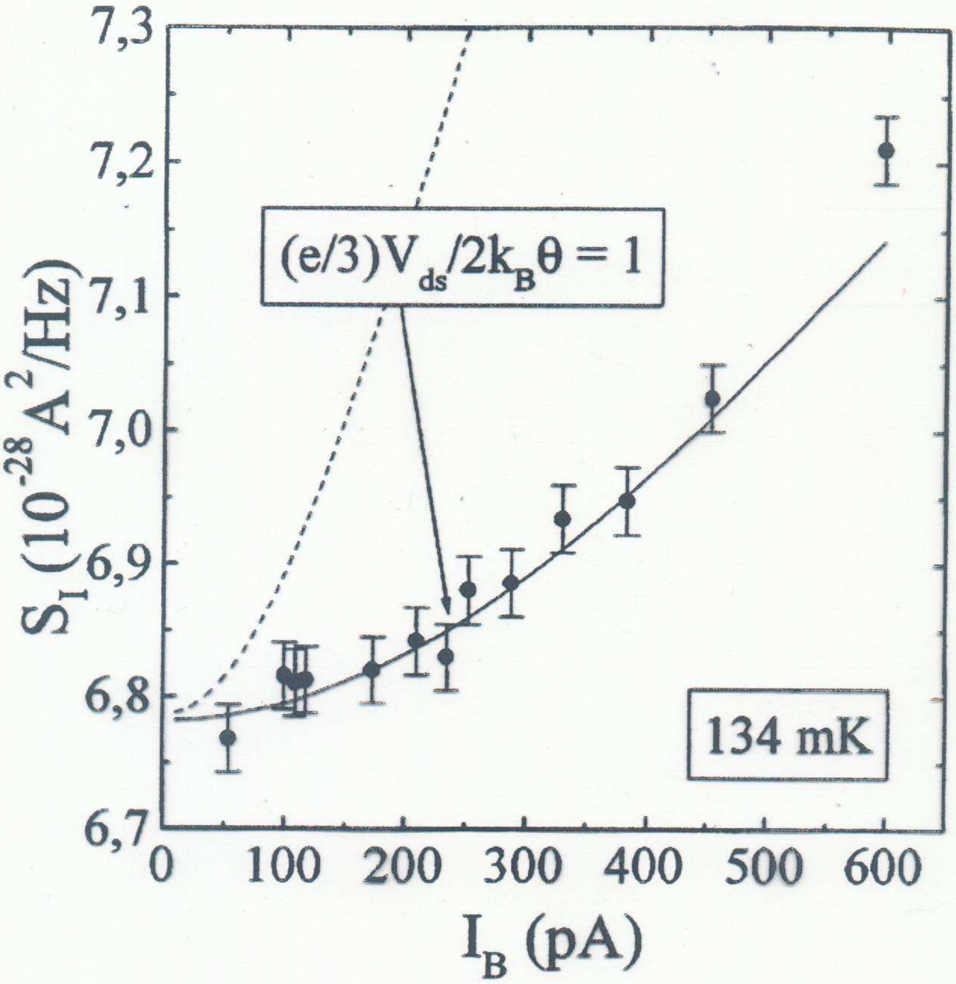


FIG. 4. Crossover from Johnson-Nyquist to shot noise. The arrow indicates the data for which $e^*V_{ds} = 2k_B\Theta$. A comparison with Eq. (2) (solid curve) and a similar expression for electrons (dotted curve) is shown.

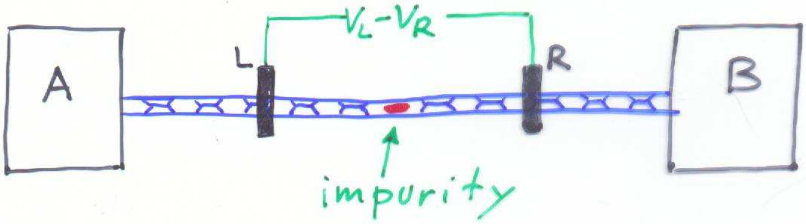
[Gattli et al., PRL ('97)]

Other instances of the Luttinger liquid

Main features:

- "anomalous" scaling laws
- spin – charge separation
- excitations with fractional charge

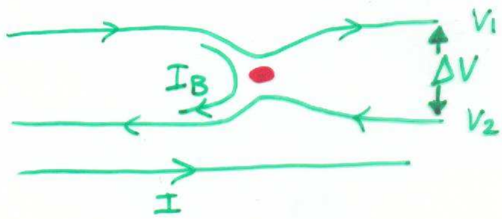
Ex: resonant tunnelling in carbon nanotubes
(M. Fisher et al. 2000)



- two bands of spinful electrons $c=4$ CFT
- Luttinger liquid behaviour demonstrated with $g \approx \nu \approx 0.2$
- expected fractionally charged excitations

Proposal: measure shot noise again: $S_I = ge I_B$

Problem: L and R chiralities not spatially separated as in QHE
$\rightarrow$ cannot measure I_B directly



Proposed solution: repeat measure with and without the impurity

Phenomenology of QHE plateaus & rotating B-E condensates

Conformal theory zoo has been very useful to model the QHE at more general plateaus

- Jain's series $\quad \nu = \dfrac{m}{mp \pm 1}$, $\quad p = 2, 4, 6, \dots$ even, $\quad m = 2, 3, \dots$

Laughlin : $m=1$, one Weyl fermion, $c=1$
general m : assume m-component fermions
$\to \widehat{SU(m)}_1 \times \widehat{U(1)}$ symmetry CFT , $c=m$

(Read; Fröhlich, Zee; ....)

Our contribution : independent derivation of these theories (+ projection of d.o.f.) from the area-preserving symmetry of the incompressible fluid (W_∞ algebra)
$\to$ W_∞ minimal models $\simeq \widehat{U(1)} \times \dfrac{\widehat{SU(m)}_1}{SU(m)}$

(A.C., C. Trugenberger, G. Zemba)
1993 —

- Paired Hall states $\quad \nu = 2 + \frac{1}{2} \quad \left(\nu = 2 + \frac{k}{kM+2}\right)$

Incompressible fluid made of paired electrons (generalized to k-clusters)
electron repulsion is softer, e.g. II Landau level,
wave function does not vanish at coincidence

→ $\hat{U}(1) \times k$-parafermion CFT $c = 1 + \frac{2(k-1)}{k+2}$

$k=2$: $\hat{U}(1) \times$ Ising : "Pfaffian state" (Moore, Read)

- new phenomenon: Non-Abelian exchange statistics

SURPRISE: these states could describe rotating BEC in the regime of very dense vortices (Cooper et al, 2001)

Landau levels	vs.	rotating vessel ω
mag. field B		$2m\omega$
Flux quantum $\frac{hc}{e}$		rotation quantum $\frac{h}{m}$
filling fraction $\nu = \frac{N_e}{N_L}$		$\nu = \frac{N}{N_V} = \frac{\#\text{ bosons}}{\#\text{ vortices}}$

Rotating BEC:

$\nu > \nu_c \sim 10$ vortex lattice (as in ^{4}He)

$\nu < \nu_c$ vortex liquid (as QHE)

- numerical simulations indicate ground states at $\nu = \frac{k}{2}$ with good overlap with k-parafermion ground states (bosonic)
- $\nu < \nu_c$ regime should be attainable experimentally
- extension to spin 1 boson state (Schoutens et al. 2002)