Ising Conformal Theory between 2 and 4 Dimensions

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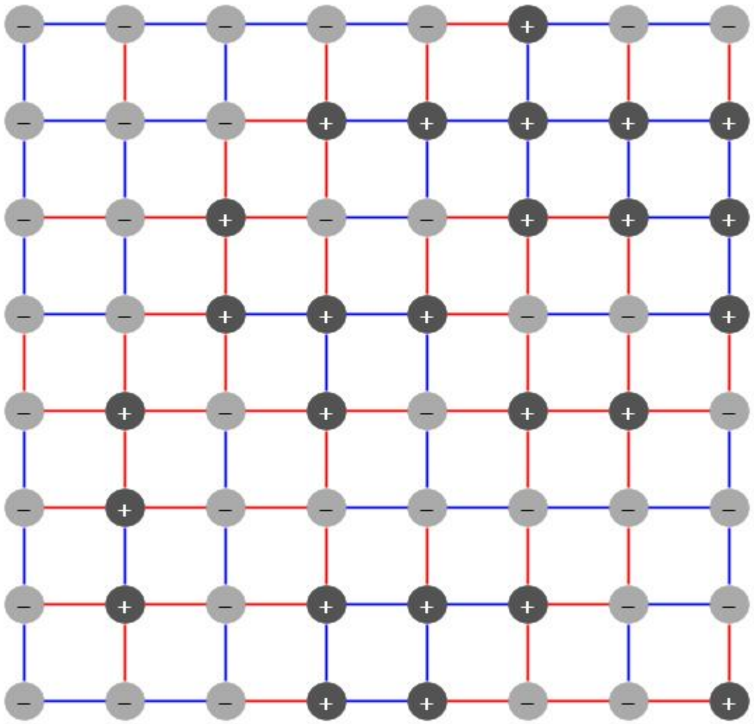
Outline

- Conformal bootstrap in varying dimension
- Precise exponents and structure constants
- Leading twists and d=2 limit
- Decoupling of states

The first love WE shall never forget

- As a Master student, I wanted to solve the 2d Ising model for $h \neq 0$



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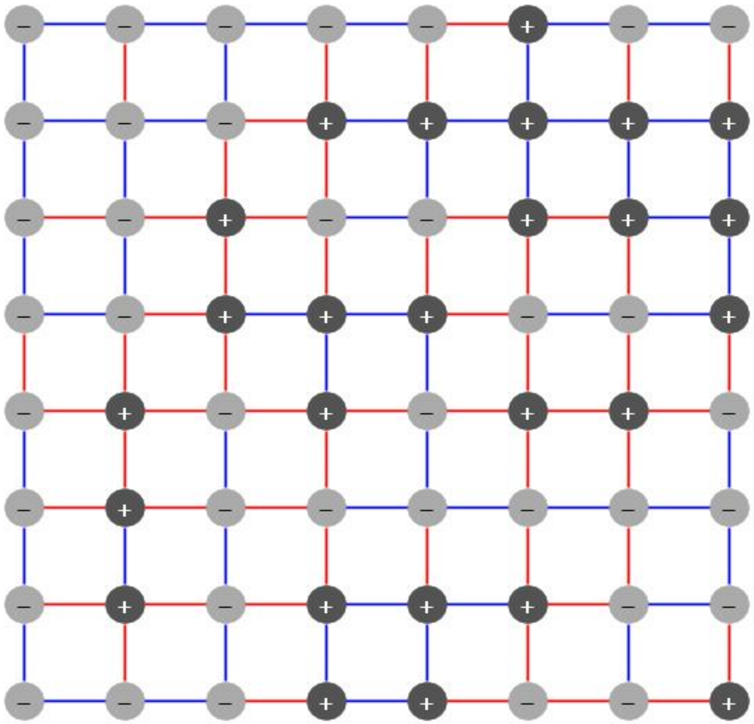

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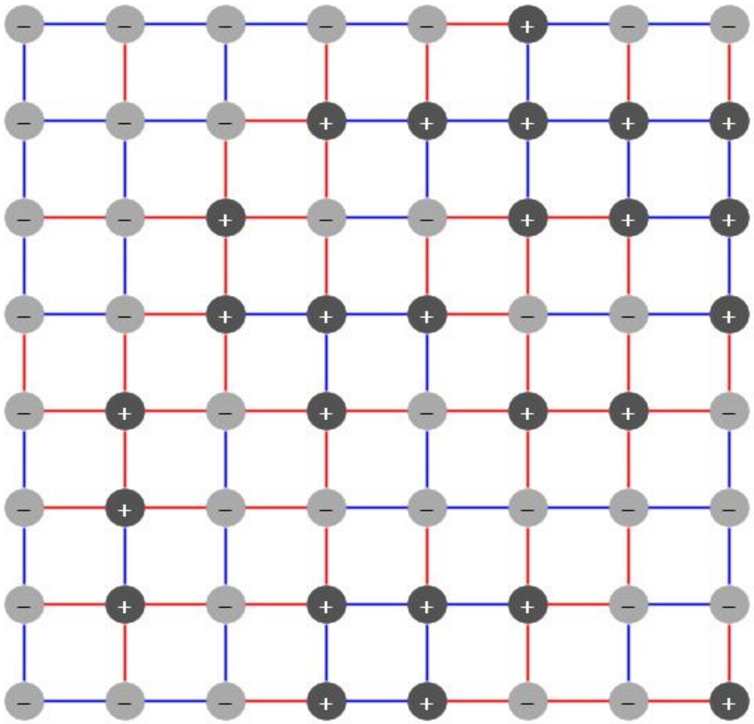

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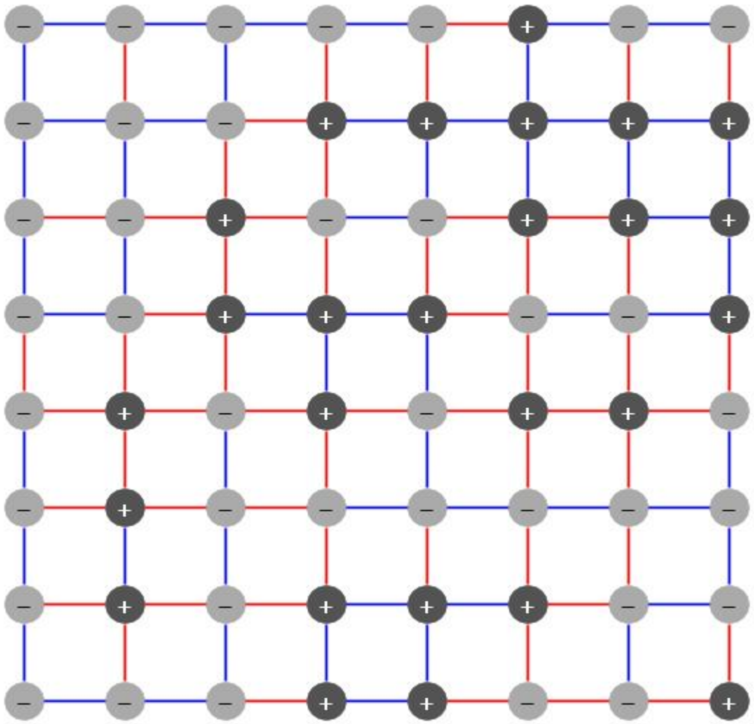
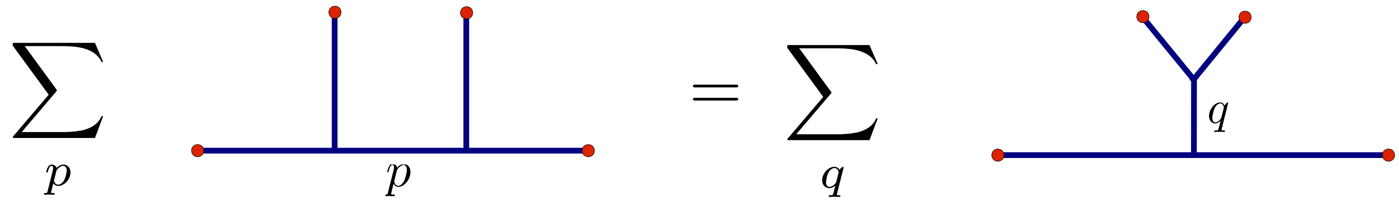
- I am sure Giuseppe had similar dreams...



Conformal bootstrap



- Original Polyakov idea, revived by Rychkov and others
- Conformal partial waves any d (different from d=2) (Dolan, Osborn '01)

$$\langle\sigma(0)\sigma(1)\sigma(\eta)\sigma(\infty)\rangle \sim \sum_{p=1}^{2} |F_p(\eta)|^2, \qquad F_p(\eta) \sim \eta^a + (1-\eta)^b \qquad d=2$$

$$\langle\sigma(0)\sigma(1)\sigma(\eta)\sigma(\infty)\rangle \sim \sum_{p'=1}^{\infty} |\widehat{F}_{p'}(\eta)|^2, \qquad \widehat{F}_{p'}(x) \sim \eta^{a+n} + \log(1-\eta) \qquad d \geq 2$$

- Needs to resum infinite logarithms, i.e. many tiny contributions
- It can be done! Routines have been implemented and are available:

SDPB (Simmons-Duffin '13), Extremal Functional (El-Showk, Paulos '13)

$$\Delta_\sigma = 0.5181489(10), \qquad \Delta_\varepsilon = 1.412625(10), \qquad d=3$$

Motivations for CFT 4>d>2

- Understand CFT in d>2 (another dream…)
- Ising CFT = singular point on the boundary of unitary bootstrap

→ reduced bootstrap

→ "minimal model"?

- The unitary boundary at d=2 corresponds to the minimal model relation

$$\Delta_{13}(c) = \frac{8}{3}\Delta_{12}(c) + \frac{2}{3}, \qquad \frac{1}{2} \leq c \leq 1$$

it extends to d>2 as

$$\Delta_{\varepsilon} \sim \frac{8}{3}\Delta_{\sigma} + \mathrm{const}(d) \qquad 2 \leq d \leq 3$$

→ let us have a look

Precise conformal data for 4>d>2

- Extend routines to $d \neq 3$ and study $13\ d$ values (single-correlator bootstrap)
- Analyze $\Delta_{\mathcal{O}}$, $f_{\sigma\sigma\mathcal{O}}$ of six low-lying fields $\sigma, \varepsilon, \varepsilon'; \; T'; \; C, C' \quad (\ell = 0, 2, 4)$
- Polynomial fit in $y = 4 - d$
- Examples:

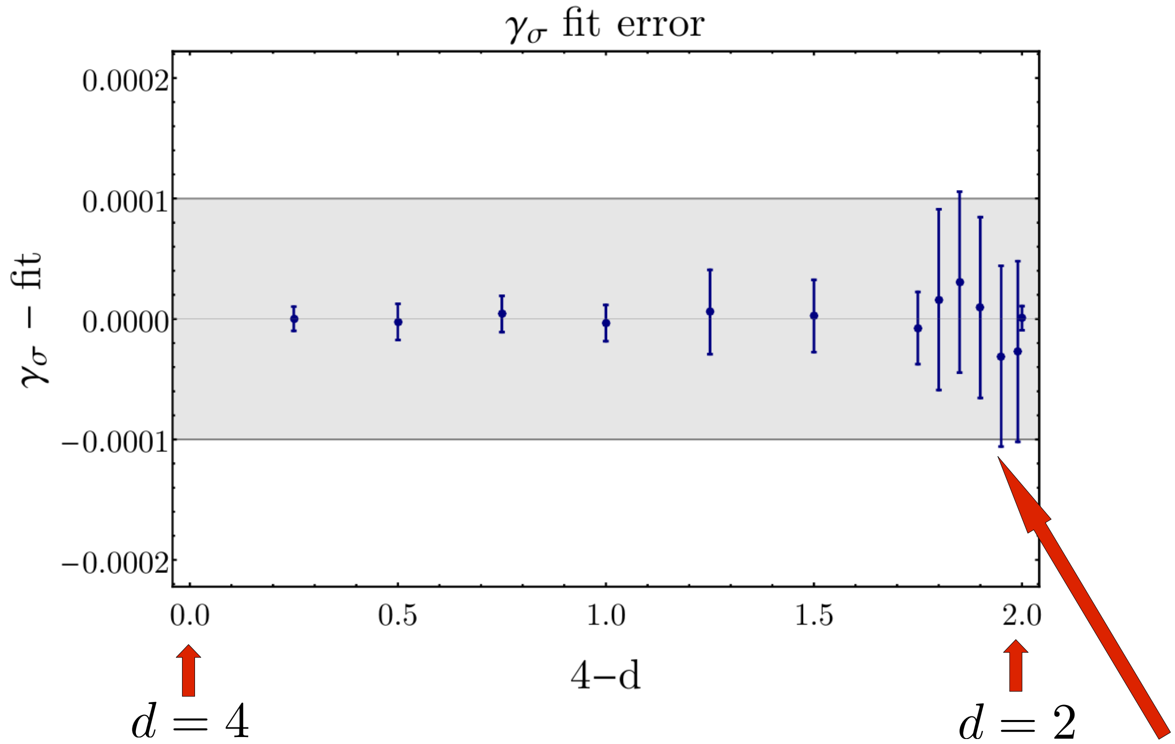
$$\Delta_\sigma = \frac{d-2}{2} + \gamma_\sigma, \qquad \Delta_\varepsilon = d - 2 + \gamma_\varepsilon$$

$$\gamma_\sigma(y) = 0.00955001 y^2 + 0.00764826 y^3 + 0.00091284 y^4$$
$$-0.00024948\ y^5 + 0.000296768 y^6, \qquad \mathrm{Err}(\gamma_\sigma) < 0.0001$$

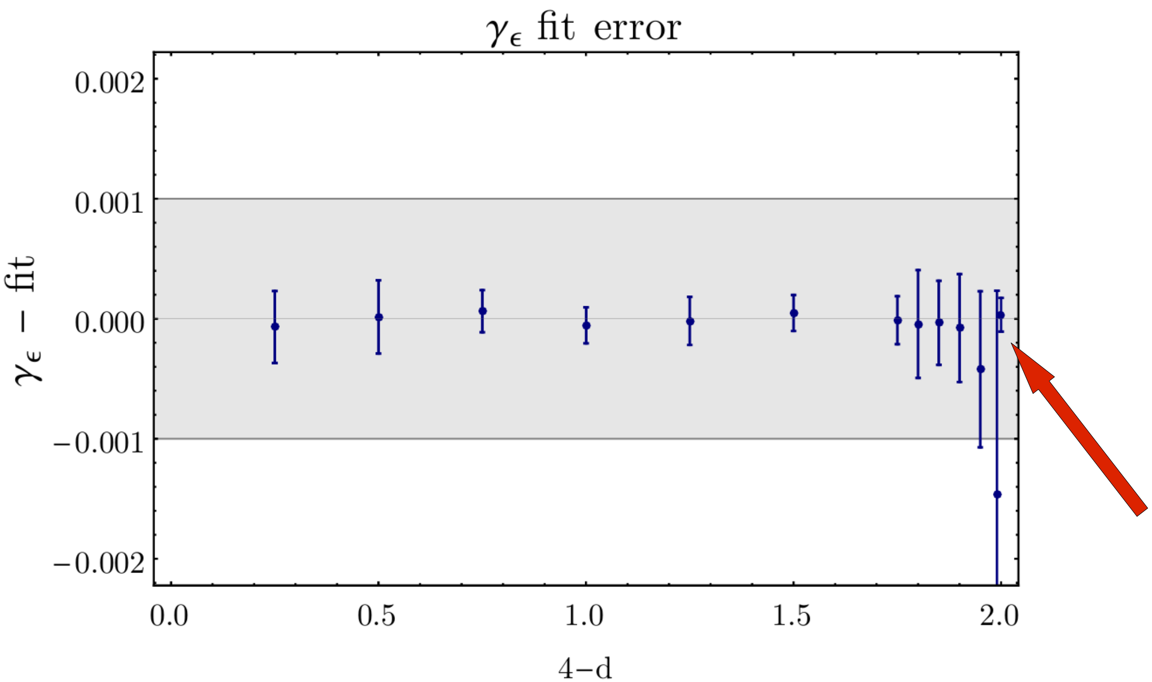
$$\gamma_\varepsilon(y) = 0.336000 y + 0.0914812 y^2 - 0.0229152 y^3 + 0.00729869 y^4$$
$$+0.000890045 y^5, \qquad \mathrm{Err}(\gamma_\varepsilon) < 0.001$$

$$f_{\sigma\sigma\varepsilon}(y) = 1.41421 - 0.235735 y - 0.164305 y^2 + 0.0631842 y^3 - 0.0371191 y^4$$
$$+0.0137454 y^5 - 0.00214024 y^6, \qquad \mathrm{Err}(f_{\sigma\sigma\varepsilon}) < 0.0002$$

overall relative error $O(10^{-3})$

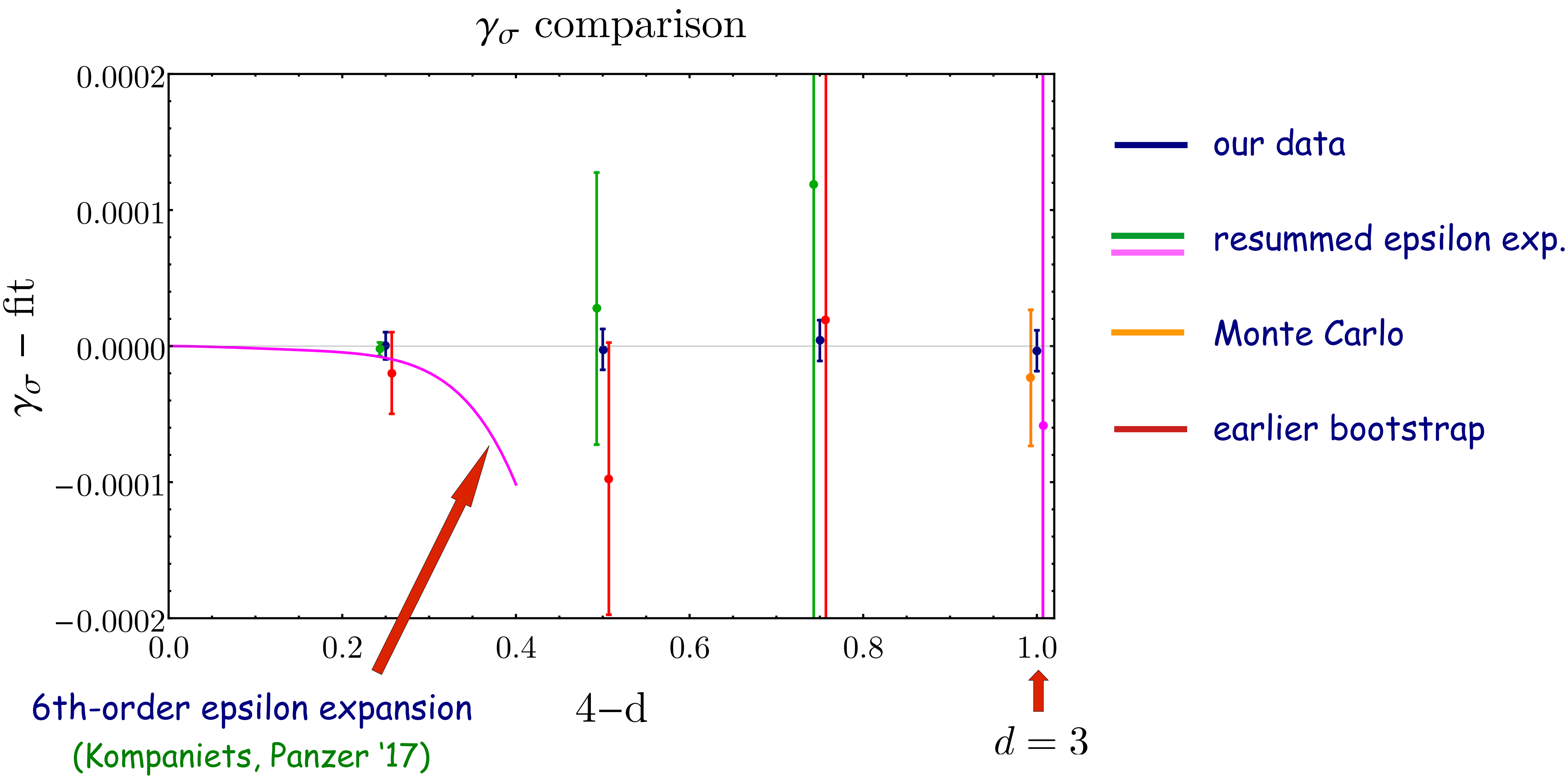


less data for 2.25>d>2



precise match at d=2

Comparison with other methods



Interest:

- test of epsilon expansion and other analytic methods
- other universality classes related to Ising(d), e.g. long-range Ising

(Behan et al. '17; Defenu et al. '17)

Leading twists and d=2 limit

- d-dependence looks smooth but actually is not
- higher part of the spectrum changes completely for d>2; numerically unstable

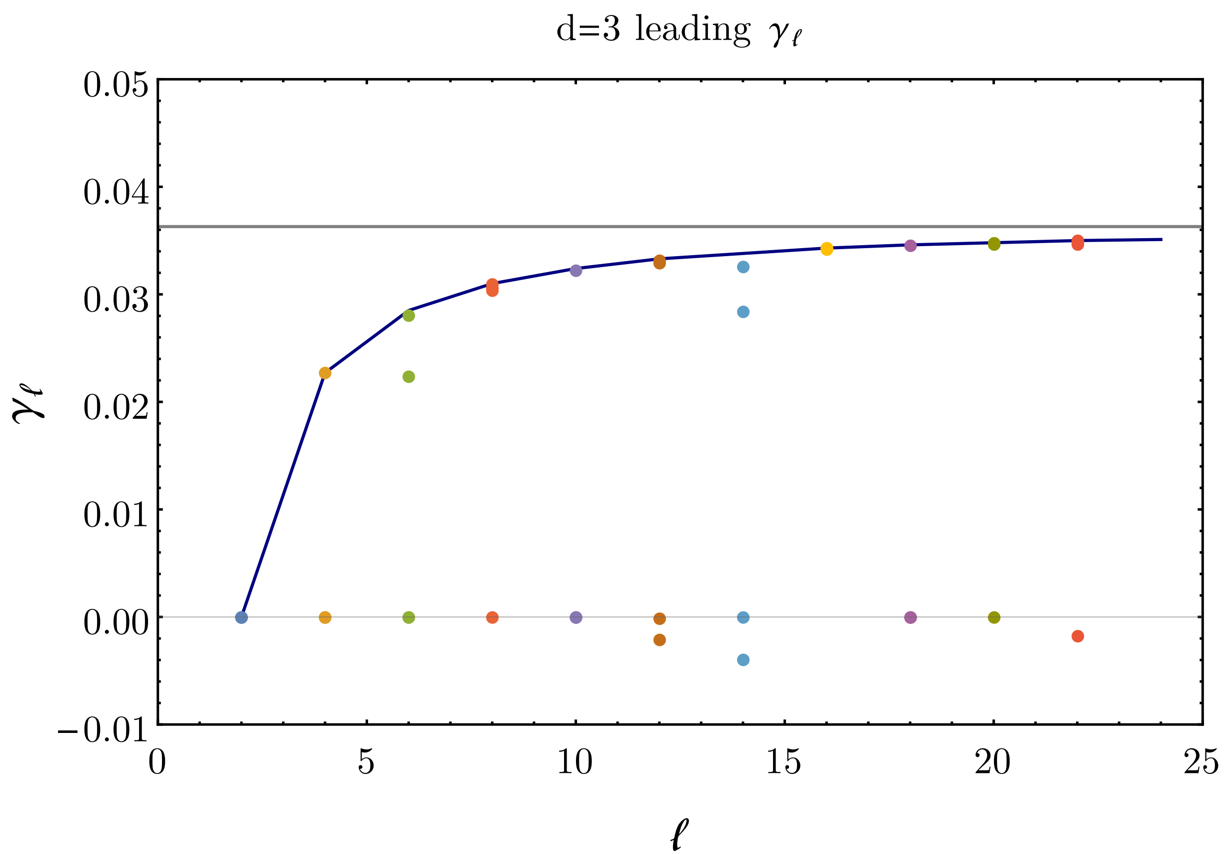
  → qualitative results
- leading twists: $\mathcal{O}_\ell$ smallest Δ_ℓ for each $\ell = 4, 6, \ldots$

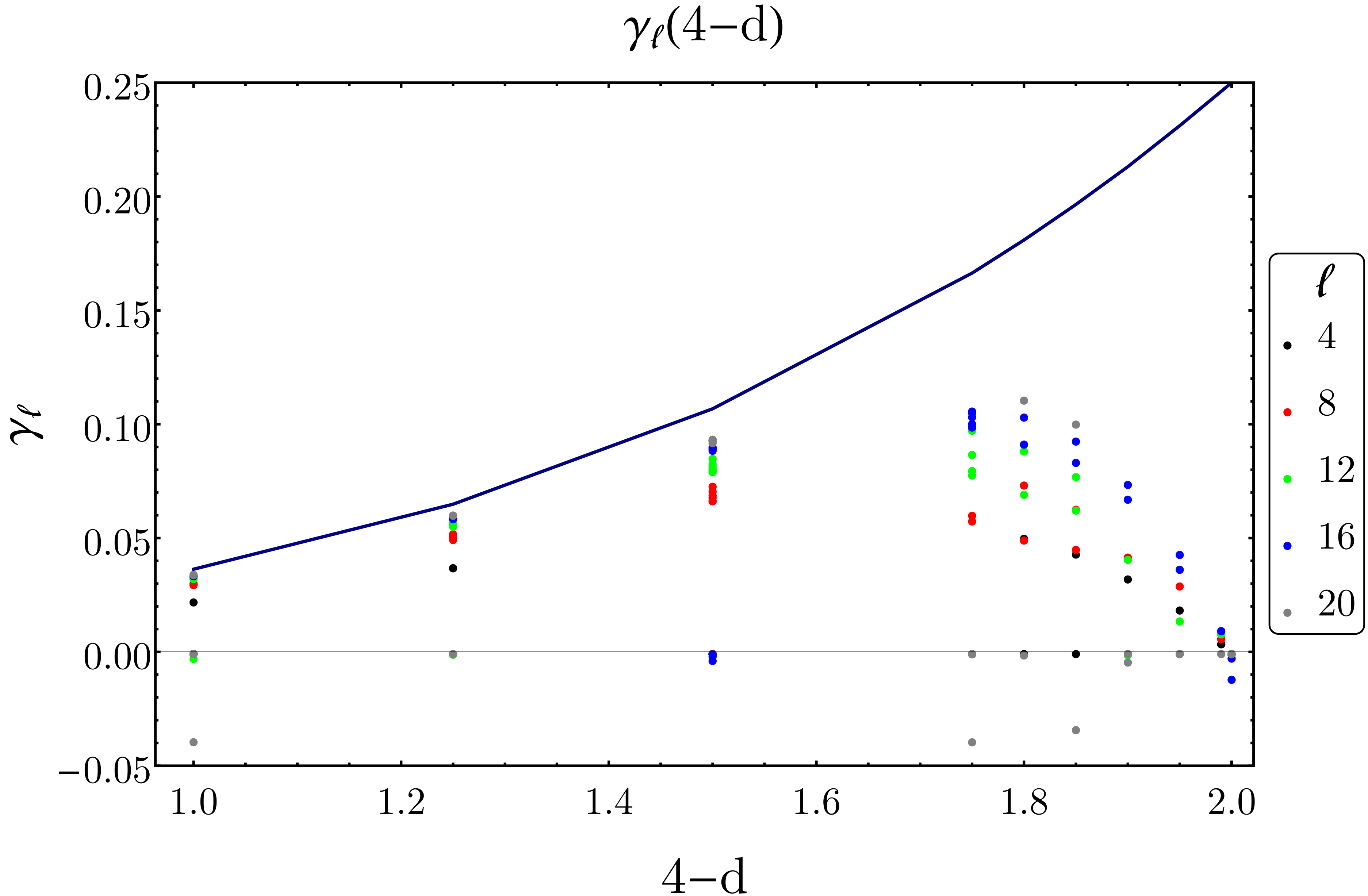
$$\Delta_\ell = \underbrace{d - 2 + \ell}_{\text{big}} + \underset{\text{small}}{\gamma_\ell}$$

$d = 3$: $\lim_{\ell\to\infty} \gamma_\ell = 2\gamma_\sigma$, monotonic in ℓ,

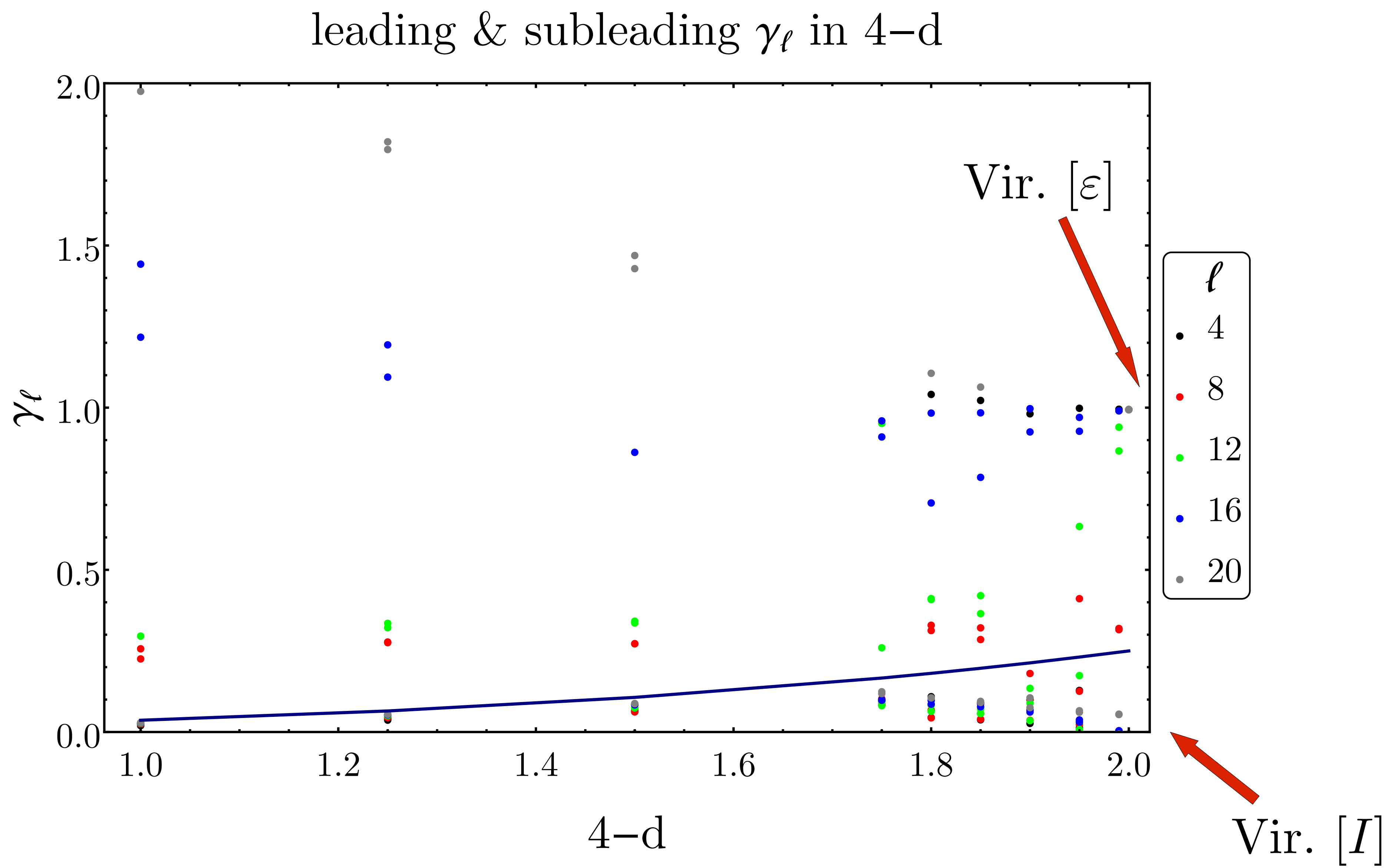
$d = 2$: $\gamma_\ell = 0 \quad \forall \ell$

↑ Virasoro Id rep.





$\gamma_\ell(4\text{−d})$
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
γ_ℓ
1.0
1.2
1.4
1.6
1.8
2.0
4−d
ℓ
4
8
12
16
20



Conclusion: d>2 behavior sets in at d ≥ 2.2 (i.e. 4 - d ≤ 1.8)

Decoupling of states

- Numerically follow the path $\Delta_\sigma(c), \Delta_\varepsilon(c)$ from Tricritical Ising to Ising at d=2
- Count quasiprimary states in both theories (A. Zamolodchikov '89)

→ necessary condition for decoupling

$$\sigma \cdot \sigma \equiv \phi_{12} \cdot \phi_{12} = \phi_{11} + \phi_{13}$$

$\boxed{1}$ decouples, well seen numerically

$\boxed{1}$ decouples, uncertain

$\boxed{2}$ decouples, unseen numerically

→ ONE clear state decoupling

at $\ell = 2$ owing to $\varepsilon \equiv \phi_{13} \sim \phi_{21}$

→ Follow it for d>2

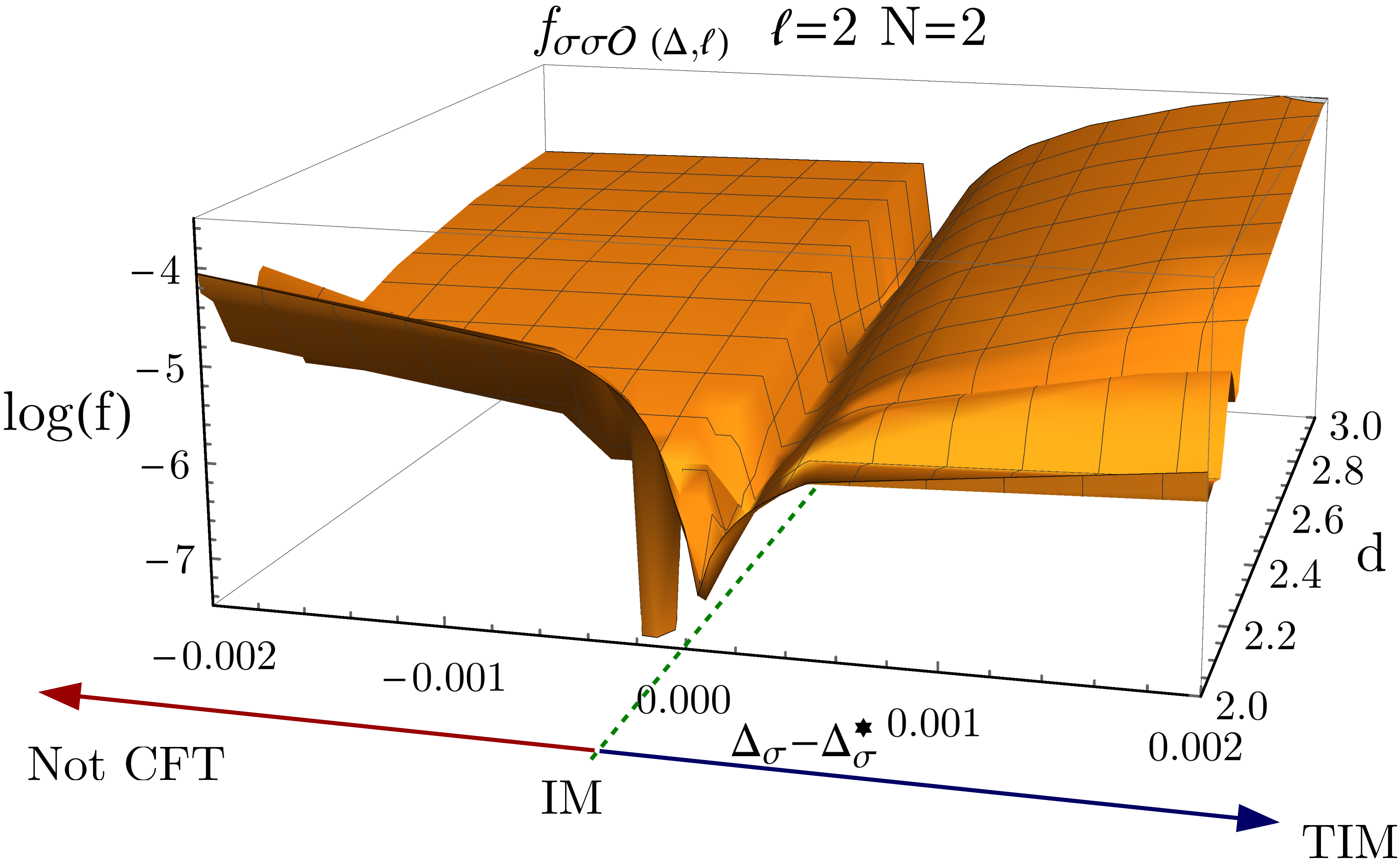
Tricritical Ising

ℓ \ Δ	1	2	3	4	5	6	7	8	9	10	11
10										4	6
8								3	4	0	0
6						2	2	0	0	3	$\boxed{4}$
4				1	2	0	0	2	$\boxed{2}$	0	0
2		1	$\boxed{1}$	0	0	1	$\boxed{2}$	0	0	2	4
0	1	0	0	1	$\boxed{1}$	0	0	1	4	0	1

Ising

ℓ \ Δ	1	2	3	4	5	6	7	8	9	10	11
10										2	2
8								2	1	0	0
6						1	1	0	0	2	0
4				1	1	0	0	1	0	0	0
2		1	0	0	0	1	0	0	0	1	1
0	1	0	0	1	0	0	0	1	1	0	0

It stays!



$f_{\sigma\sigma\mathcal{O}\,(\Delta,\ell)}$ ℓ=2 N=2
log(f)
−4
−5
−6
−7
−0.002
−0.001
0.000
0.001
0.002
$\Delta_\sigma - \Delta_\sigma^\star$
d
2.0
2.2
2.4
2.6
2.8
3.0
Not CFT
IM
TIM

Conclusions

- Ising universality class as a function of d can be useful
- Leading twists: a glimpse into an unknown world, the $d > 2$ conformal spectrum
- Ising "reduced bootstrap" is yet a mystery... or a red herring...

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with great respect and friendship,

Andrea