Conformal Field Theory of Composite Fermions in the QHE

Andrea Cappelli
(INFN and Physics Dept., Florence)

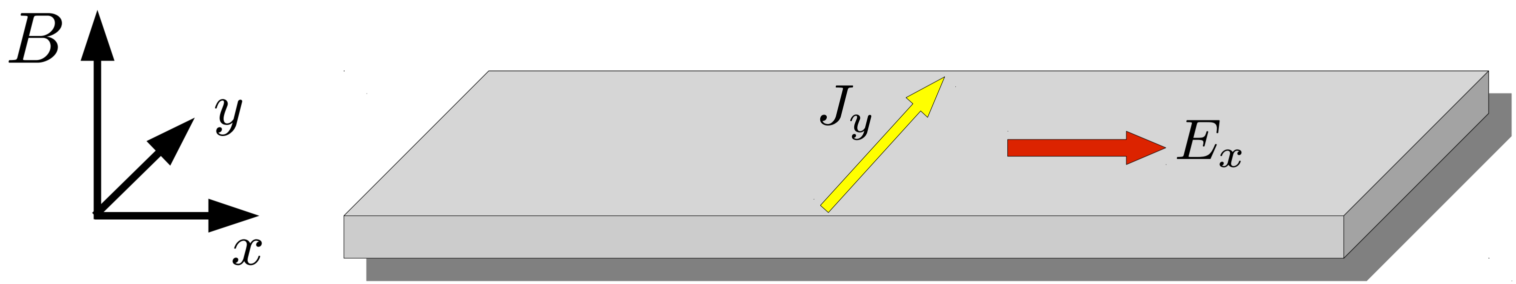
Outline

- Introduction: wave functions, edge excitations and CFT
- CFT for Jain wfs: Hansson et al. results
- CFT for Jain wfs: W-infinity minimal models
- independent derivation of Jain wfs from symmetry arguments
- CFT suggests non-Abelian statistics of quasi-holes

Quantum Hall Effect

- 2 dim electron gas at low temperature T ~ 10 mK

  and high magnetic field B ~ 10 Tesla



- Conductance tensor $J_i = \sigma_{ij} E_j, \quad \sigma_{ij} = R^{-1}_{ij}, \qquad i, j = x, y$
- Plateaux: $\sigma_{xx} = 0, \ R_{xx} = 0$ no Ohmic conduction → gap

  $$\sigma_{xy} = R^{-1}_{xy} = \frac{e^2}{h}\nu, \quad \nu = 1(\pm 10^{-8}), 2, 3, \ldots \frac{1}{3}, \frac{2}{5}, \ldots, \frac{5}{2},$$

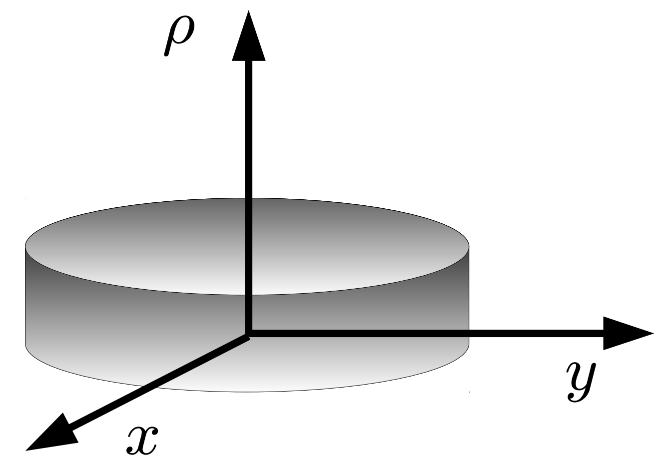
- High precision & universality
- Uniform density ground state: $\rho_o = \frac{eB}{hc}\nu$

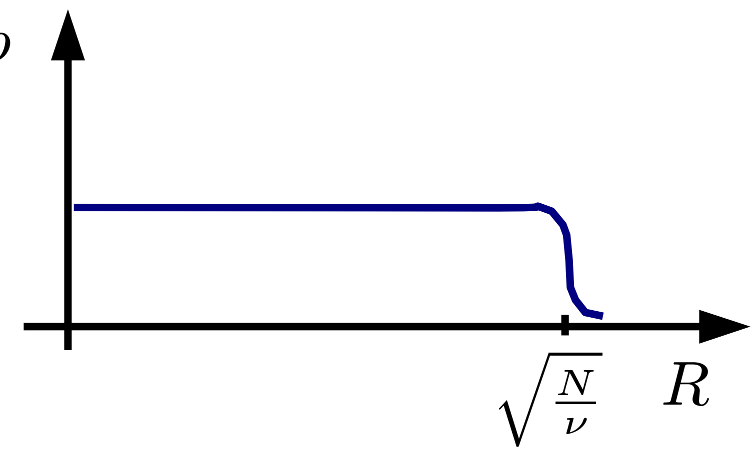
Incompressible fluid

Laughlin's quantum incompressible fluid

Electrons form a droplet of fluid:

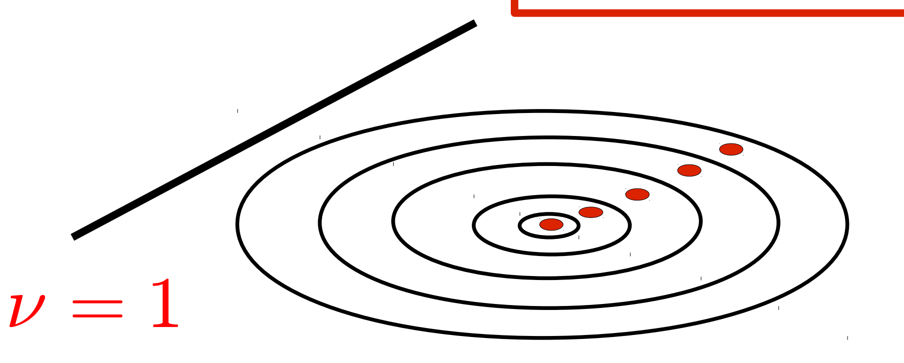
- incompressible = gap
- fluid = $\rho(x,y) = \rho_o = \text{const.}$

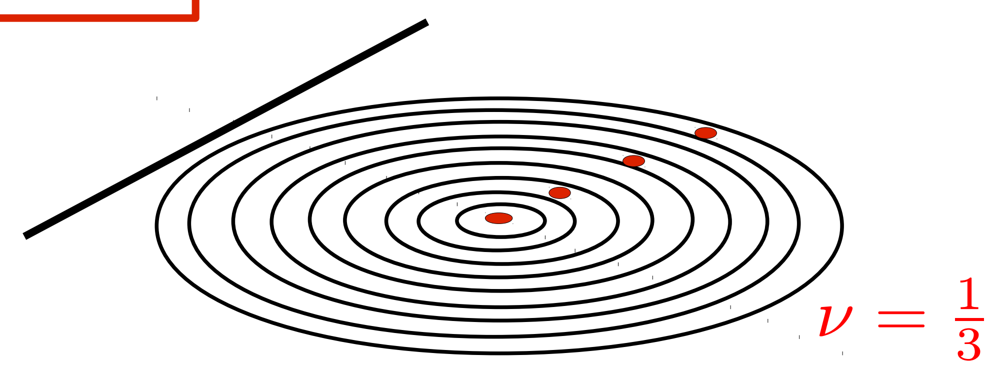




degenerate orbitals = # quantum fluxes $\quad \mathcal{D}_A = \frac{BA}{\Phi_o}, \quad \Phi_o = \frac{hc}{e}$

filling fraction: $\boxed{\nu = \frac{N}{\mathcal{D}_A} = 1, 2, \ldots \frac{1}{3}, \frac{1}{5}, \ldots}$ density for quantum mech.





Laughlin's wave function

$$\Psi_{gs}(z_1, z_2, \ldots, z_N) = \prod_{i<j} (z_i - z_j)^{2k+1} \, e^{-\sum |z_i|^2/2}$$

$$\nu = \frac{1}{2k+1} = 1, \frac{1}{3}, \frac{1}{5}, \ldots$$

- $\nu = 1$ filled Landau level: obvious gap $\omega = \frac{eB}{mc} \gg kT$
- $\nu = \frac{1}{3}$ non-perturbative gap due to Coulomb interaction

→ effective theories

- quasi-holes = vortices $\Psi_\eta = \prod_i (\eta - z_i) \Psi_{gs}$

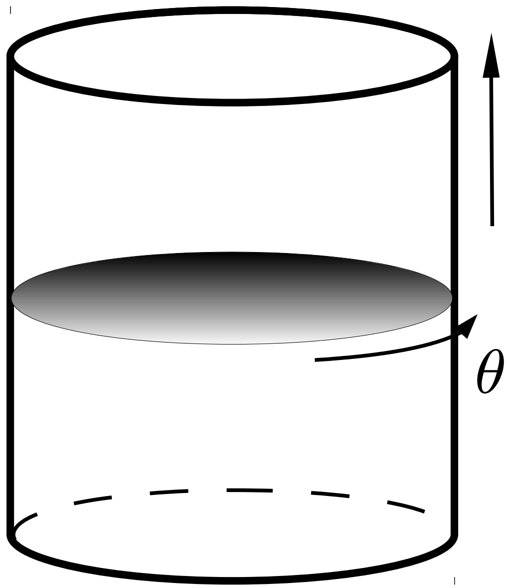
$$\Psi_{\eta_1,\eta_2} = (\eta_1 - \eta_2)^{\frac{1}{2k+1}} \prod_i (\eta_1 - z_i)(\eta_2 - z_i) \Psi_{gs}$$

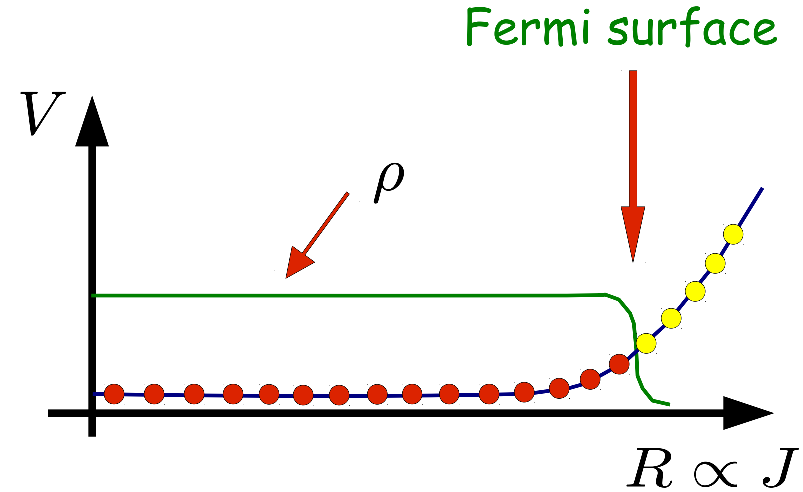
→ fractional charge $Q = \frac{e}{2k+1}$ & statistics $\frac{\theta}{\pi} = \frac{1}{2k+1}$

Anyons vortices with long-range topological correlations

Conformal field theory of edge excitations

The edge of the droplet can fluctuate: edge waves are massless

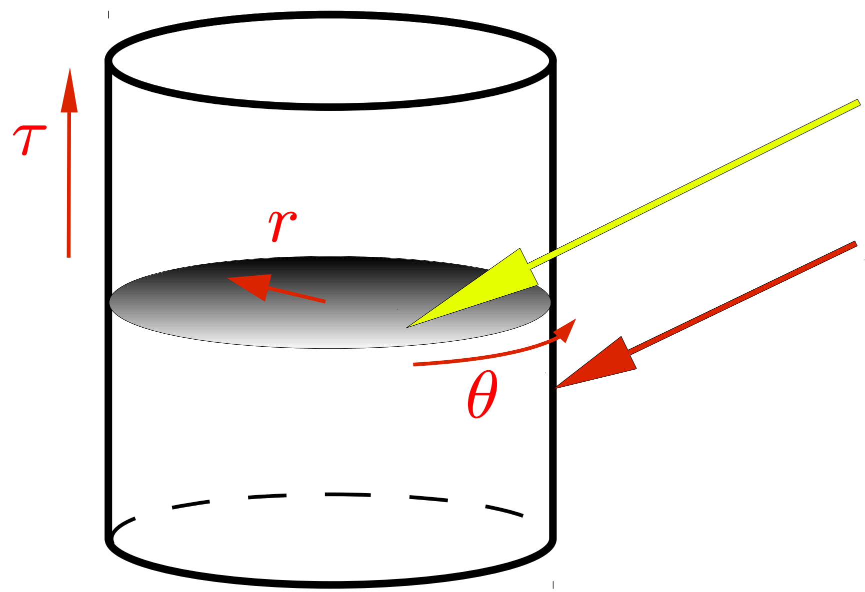




- edge ~ Fermi surface: linearize energy $\varepsilon(k) = \frac{v}{R}(k - k_F), \quad k = 0, 1, \ldots$
- relativistic field theory in 1+1 dimensions, chiral (X.G.Wen '89)

→ chiral compactified c=1 CFT (chiral Luttinger liquid)

CFT descriptions of QHE



$z = re^{i\theta}$ plane (bulk excit)

$\zeta = e^{\tau} e^{i\theta}$ cylinder (edge excit)

$\langle\ \phi_1 \quad \phi_2\ \rangle_{CFT}$

$(z_1 - z_2)^{\alpha^2}$ anyon wavefunction

$(\zeta_1 - \zeta_2)^{\alpha^2}$ edge-excit. correlator

➡ wavefunctions: spectrum of anyons and braiding matrices

➡ edge correlators: physics of conduction experiments

- equivalence of descriptions: analytic continuation from the circle,

  use map CFT ⟷ Chern-Simons theory in 2+1 dim

- general CFT is U(1) x neutral

Non-Abelian fractional statistics

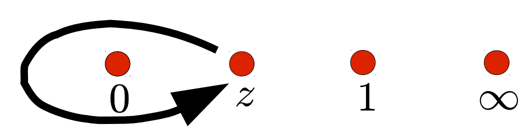
- $\nu = \frac{5}{2}$ described by Moore-Read "Pfaffian state" ~ Ising CFT x U(1)
- Ising fields: I identity, ψ Majorana = electron, σ spin = anyon
- fusion rules:
  - $\psi \cdot \psi = I$ 2 electrons fuse into a bosonic bound state
  - $\sigma \cdot \sigma = I + \psi$ 2 channels of fusion = 2 conformal blocks

$$\langle \sigma(0)\sigma(z)\sigma(1)\sigma(\infty)\rangle = a_1 F_1(z) + a_2 F_2(z)$$

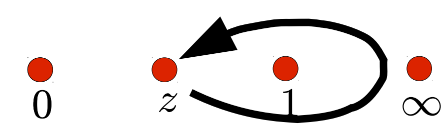
→ state of 4 anyons is two-fold degenerate (Moore, Read '91)

- statistics of anyons ~ analytic continuation → 2x2 matrix

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}(ze^{i2\pi}) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}(z)$$



$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}(1+(z-1)e^{i2\pi}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}(z)$$



(all CFT redone for Q. Computation: M. Freedman, Kitaev, Nayak, Slingerland,....)

Jain composite fermion

$$\Psi_{\nu=\frac{1}{p+1}} = \prod_{i<j} z_{ij}^{p} \prod_{i<j} z_{ij} = \prod_{i<j} z_{ij}^{p} \; \Psi_{\nu=1}, \qquad p \text{ even} \qquad z_{ij} = z_i - z_j$$

- Correspondence FQHE ⟷ IQHE

$$\frac{N_\Phi}{N_e} = \frac{1}{\nu} = p+1 \qquad\qquad \frac{1}{\nu^*} = 1$$

$$B \qquad\qquad B^* = B - p\,\rho\,\Phi_o$$

- generalize to n filled Landau levels

$$\frac{1}{\nu} = p + \frac{1}{n} \qquad\qquad \frac{1}{\nu^*} = \frac{1}{n}$$

$$\Psi_{\nu=\frac{n}{np+1}} = \mathcal{P}_{LLL} \prod_{i<j} z_{ij}^{p} \; \Psi_{\nu^*=n}\left(\bar{z}_i, z_i\right)$$

- composite fermion: quasiparticle feeling the reduced B^*
- many experimental confirmations + mean field theory (Lopez, Fradkin;....)
- $\Psi_{\nu=\frac{n}{np+1}}$ written directly in LLL using projection $\bar{z}_i \to \partial_{z_i}$ in $\Psi_{\nu^*=n}$

(Jain, Kamilla '97)

CFT for Jain: Hansson et al. ('07-'11)

$$\Psi_{\nu=\frac{2}{2p+1}} = \mathcal{A}\left[\prod^{N/2} w_{ij}^{p+1}\, \partial_{z_1}\cdots\partial_{z_{N/2}} \prod^{N/2} z_{ij}^{p+1} \prod^{N/2} (z_i - w_j)^p\right]$$

$$\frac{1}{\nu} = p + \frac{1}{2}$$

- result based on non-trivial algebraic identites
- recover Abelian two-component edge theory

$$K = \begin{pmatrix} p+1 & p \\ p & p+1 \end{pmatrix}$$

(Wen, Zee; Read,....)

$$\Psi_{\nu=\frac{2}{2p+1}} = \mathcal{A}\left[\left\langle(\partial_{z_1}V_+)\cdots(\partial_{z_{N/2}}V_+)\,V_-\cdots V_-\right\rangle\right]$$

$$V_\pm = e^{i\sqrt{p+\frac{1}{2}}\,\varphi}\, e^{\pm i\frac{1}{\sqrt{2}}\phi}$$

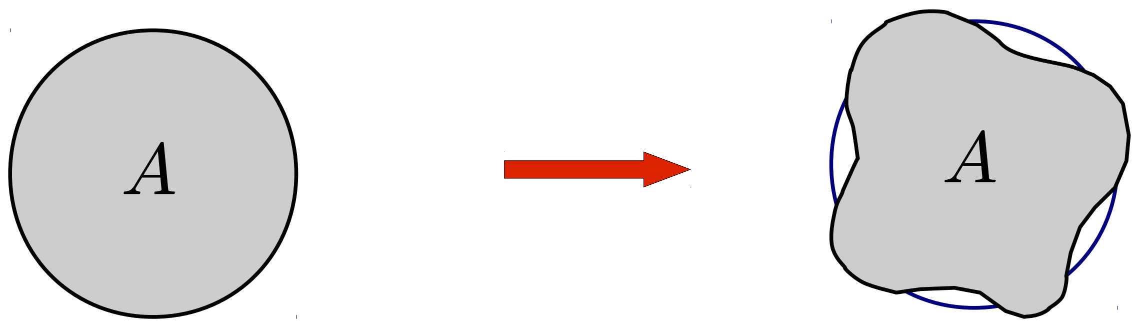
charged neutral

- but there is more:
  - $\mathcal{A}$: two fermions V_+, V_- ➡ one fermion
  - descendant fields needed for non-vanishing result, yield correct "shift"
- Next: find improved CFT that complete the derivation

W-infinity symmetry

Area-preserving diffeomorphisms of incompressible fluid

$$\int d^2x\ \rho(x) = N = \rho_o A \quad \longrightarrow \quad \underline{A = \text{constant}}$$



- W-infinity symmetry can be implemented in the edge CFT
- CFT with higher currents characteristic of 1d fermions (+ bosonization)

$$W^k =: \bar{F}\,(\partial_z)^k\,F :, \qquad W^0 = \bar{F}F = J, \qquad W^1 = T =: J^2 :, \qquad W^2 =: J^3 :, \quad \cdots$$

- representations completely known ⟶ classification (V.Kac, A. Radul '92)
- $c = n$ generically reproduce Abelian theories $\widehat{U(1)}^n$ with K matrices
- but special representations for enhanced symmetry $\widehat{U(1)} \times \widehat{SU(n)}_1$

W-infinity minimal models

- repres. with enhanced symmetry are degenerate and should be projected:
  → W_∞ minimal models (A.C., Trugenberger, Zemba '93-'99)

$$\widehat{U(1)}^{n} \longrightarrow \widehat{U(1)} \times \widehat{SU(n)}_1 \longrightarrow \widehat{U(1)} \times \frac{\widehat{SU(n)}_1}{SU(n)} = \widehat{U(1)} \times \mathcal{W}_n$$

- these edge theories reproduce Jain fillings, $\nu = \dfrac{n}{p\ n \pm 1}$
  with usual K matrices for charge and statistics
- extra projection of $SU(n)$ amounts to keeping edge excitations completely symmetric w.r.t. layer exchanges:
  - single electron excitation
  - reduced multiplicities of edge states
  - non-Abelian statistics of quasi-particles & electron (see later)

Ex: c=2 minimal model

$$\widehat{U(1)} \times \widehat{SU(2)}_1 \longrightarrow \widehat{U(1)} \times \frac{\widehat{SU(2)_1}}{SU(2)} = \widehat{U(1)} \times \mathrm{Vir}$$

$$c = 2, \quad \frac{1}{\nu} = p + \frac{1}{2}$$

Vir = SU(2) Casimir subalgebra

- take edge excitations symmetric w.r.t. two layers only
- neutral part is described by the Virasoro minimal model for $c \to 1$
- fields characterized by dimension $h = \frac{k^2}{4}$ i.e. total spin $s = \frac{k}{2}$; NO s_z
- electron has $s = \frac{1}{2}$
- identify two vertex operators by Dotsenko-Fateev screening operators $Q_\pm$

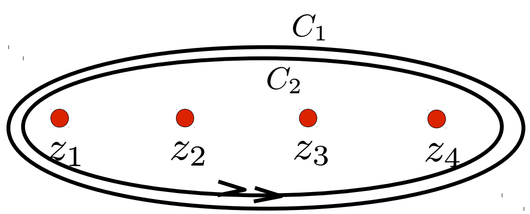
$$V_\pm = e^{\pm \frac{i}{\sqrt{2}}\phi}, \quad s_z = \pm\frac{1}{2}, \quad V_- \sim V_+ = Q_+ V_-, \quad Q_+ = J_0^+ = \oint du J^+(u)$$

(Felder)

Derivation of Jain wf in Hansson et al. form

- use W-infinity minimal models to describe ground state wf
- 4-el. wf has two channels, $\{\frac{1}{2}\} \times \{\frac{1}{2}\} = \{0\} + \{1\}$, given by choices of C_i

$$\Psi = \left\langle \oint_{C_1} J^+ \oint_{C_2} J^+ \; V_-(z_1) V_-(z_2) V_-(z_3) V_-(z_4) \right\rangle$$



- impose antisymm of electrons ➡ $\Psi = 0$
- consider descendant with same charge: $J_0^+ \to J_{-1}^+ = L_{-1} J_0^+$, $J_{-1}^+ V_- \sim \partial_z V_+$

$$\Psi' = \left\langle J_{-1}^+ V_-(z_1) \; J_{-1}^+ V_-(z_2) \; V_-(z_3) \, V_-(z_4) \right\rangle + \mathrm{perm} = \Psi_{\mathrm{Jain}}$$

➡ Underlying theory of Jain wf is W-infinity minimal model + Fermi statistics for electrons

➡ Indipendent, exact derivation of Jain state from symmetry principles ➡ universality, robustness, etc.

Jain wf vs. Pfaffian wf

$$\Psi_{\nu=\frac{2}{2p+1}} = \mathcal{A}\left[\prod^{N/2} w_{ij}^{p+1}\, \partial_{z_1}\cdots\partial_{z_{N/2}} \prod^{N/2} z_{ij}^{p+1} \prod^{N/2}(z_i - w_j)^p\right], \qquad \frac{1}{\nu} = p + \frac{1}{2}$$

- reminds of Paffian state in Abelian version (A.C, Georgiev, Todorov '01)

$$\Psi_{\mathrm{Pfaff}} = \mathcal{A}\left[\prod^{N/2} w_{ij}^{M+2} \prod^{N/2} z_{ij}^{M+2} \prod^{N/2}(z_i - w_j)^M\right], \quad M \text{ odd}, \quad K = \begin{pmatrix} M+2 & M \\ M & M+2 \end{pmatrix}$$

- same vanishing behaviour:

$$\Psi \sim z_{12}^{p-1}\left(z_{13}^2\ z_{14}^2\cdots\right), \qquad \frac{1}{\nu} = p + \frac{1}{2}$$

$$\Psi \sim \left(z_{12}\, z_{13}\, z_{23}\right)^{p-1}\left(z_{14}^2\ z_{15}^2\cdots\right), \qquad \frac{1}{\nu} = p + \frac{1}{3}$$

➡ $p = 1$ Jain is excited state of the $M = 0$ Pfaffian

- same pairing?
- fractional statistics?

cf. Simon, Rezayi, Cooper '07;
Regnault, Bernevig, Haldane '09

Pfaffian state & excitations

$$\Psi_{\mathrm{Pfaff}} = \mathcal{S}\left[\prod^{N/2} w_{ij}^2 \prod^{N/2} z_{ij}^2\right] = \prod^{N} z_{ij}\ \mathrm{Pf}\left(\frac{1}{z_{ij}}\right), \qquad M = 0,$$

- projection = symmetric layers, $\chi_1 \pm \chi_2 \;\rightarrow\; \chi_1$ Weyl → Majorana
- ground state & excitations are singlets
- distinguishable excit. (Abelian) → identical excit. (non-Abelian)
- 3-body pseudo-potential (Read, Rezayi '99)

Jain state & excitations

- ground state: $SU(2)$ singlet (up to short-distance deformation $V \rightarrow \partial V$)
- excitations:
  - → distinguishable → Abelian
  - → also singlets (W_∞ irrep.) → non-Abelian

Non-Abelian statistics of Jain quasiholes

- smallest q-hole, e.g. $Q = \frac{1}{5}$ at $\nu = \frac{2}{5}$, has neutral part $s = \frac{1}{2}$:

  ➡ two components $s_z = \pm\frac{1}{2}$ identified by the projection $H_+ \sim J_0^+ \, H_-$

  i.e. q-holes in two layers are symmetrized

- fusion $\begin{cases} H_\pm H_\pm \sim H_{s=1} \\ H_+ H_- \sim H_{s=0} = I \end{cases}$ ➡ $HH \sim I + H_{s=1}$ non-Abelian statistics

- first non-trivial case is 4 q-holes: three independent states

$$\Psi_{(12,34)} = \mathcal{A}_{z_i} \left\langle H_+(\eta_1) H_+(\eta_2) H_-(\eta_3) H_-(\eta_4) \prod V_e(z_i) \right\rangle + (+ \leftrightarrow -)$$

$$\Psi_{(13,24)} = \mathcal{A}_{z_i} \left\langle (+-+-) \right\rangle , \qquad \Psi_{(14,23)} = \mathcal{A}_{z_i} \left\langle (+--+) \right\rangle$$

- they trasform among themselves under monodromy

  ➡ multidimensional representation

projection: permutation of different excit. ➡ exchange of identical excit.

Remarks

- $2k$ quasiholes have quantum dimension $d_k \sim \frac{2^{2k}}{\sqrt{k}}$ (not Rational CFT)
- other Jain q-holes: $\langle(+++++)\rangle$, odd no., antisymm combinations, are projected out in the W_∞ minimal theory.
- Jain quasi-particles & hierarchy (after Hansson et al.) also fit in
- edge is consistently non-Abelian (long-distance physics)

But

- energetics of W_∞ projection not understood (in the bulk)
- entaglement spectrum does not seem to show projection

Conclusions

- CFT + W-infinity symmetry rederive Jain states

  → independently of composite fermion picture ←

- Jain states are consistent & universal
- same CFT hints at non-Abelian q-hole excitations
- open problems:
  - investigate space of states and energy spectrum
  - relation to Pfaffian and Gaffnian states and their CFTs
- experimental tests:
  - thermopower, if measure can be extended to higher B
  - puzzles in known experiments?