Surface excitations of 3d TI: conformal invariance, self-duality and bosonization

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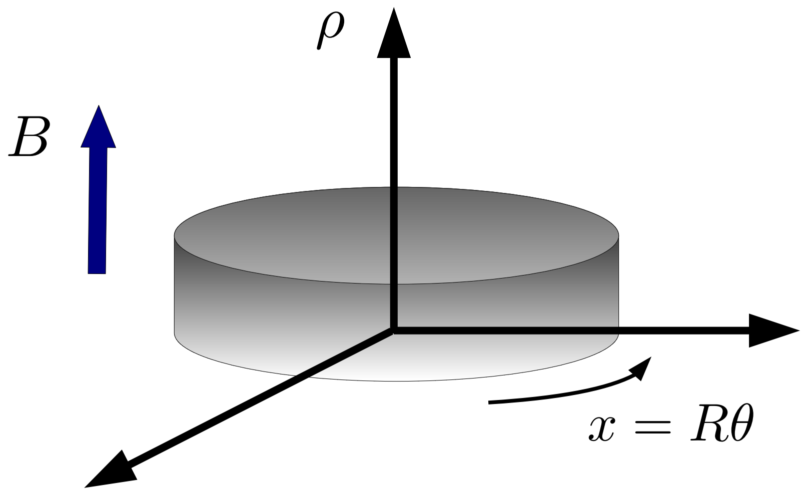
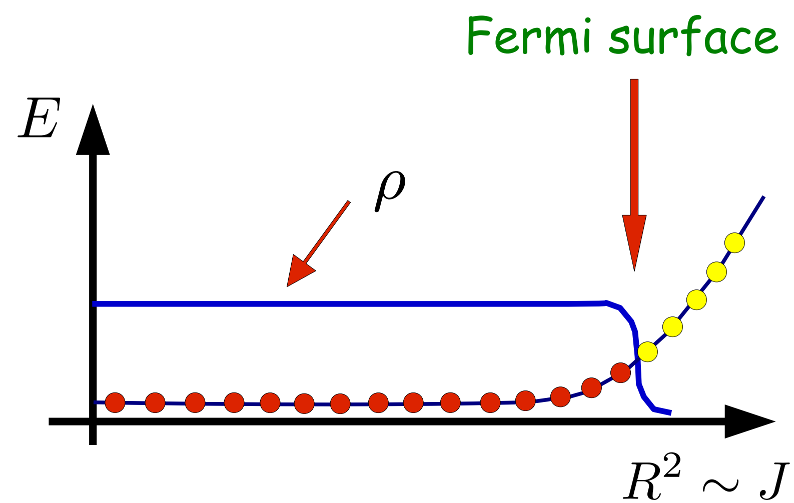
Outline

- Topological states of matter: bulk and boundary
- Surface excitations: fermionic and bosonic descriptions
- Quantization of the (2+1)d non-local Abelian gauge theory
- Self-duality and conformal invariance
- Bosonization from the bulk-boundary correspondence

Quantum Hall effect: edge fermions

Filled Landau level: bulk gap $\omega \propto B \gg kT$, but edge can fluctuate





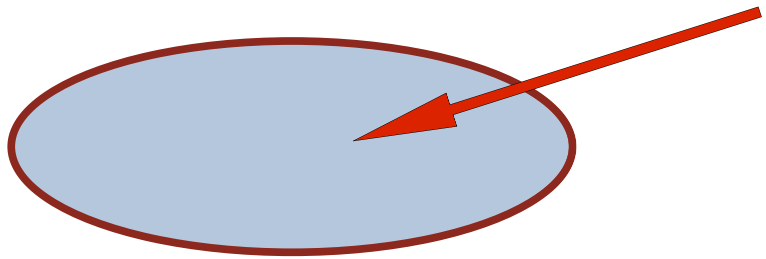
edge ~ Fermi surface: linearize energy $\varepsilon(k) = \frac{v}{R}(k - k_F), \quad k \in \mathbb{Z}$

→ chiral massless fermion in (1+1) dimensions $\psi(r, \theta, t)|_{r=R}$

→ fractional filling → interacting fermion → bosonization

→ conformal field theory (Luttinger liquid)

Bosonic description: bulk



$$j^\mu = \frac{1}{2\pi}\varepsilon^{\mu\nu\rho}\partial_\nu a_\rho$$ matter fluctuations

$$a = a_\mu dx^\mu$$ hydrodynamic gauge field

- bulk theory is topological at energies below the gap, no local degrees of freedom

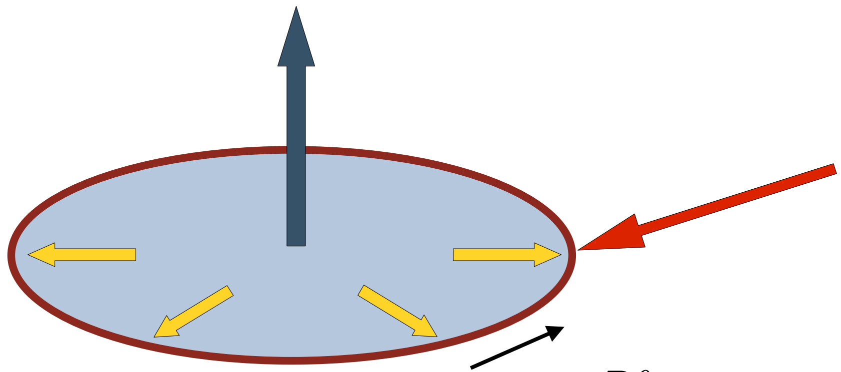
$$S_{\mathrm{bulk}}[a, A] = \int \frac{k}{4\pi} a da + \frac{1}{2\pi} a dA \quad \Rightarrow \quad S_{\mathrm{ind}}[A] = \frac{1}{4\pi k}\int A dA$$

- Hall current

$$J^i = \frac{\delta S_{\mathrm{ind}}}{\delta A_i} = \frac{\nu}{2\pi}\varepsilon^{ij}\mathcal{E}^j \qquad \nu = \frac{1}{k} = 1, \frac{1}{3}, \frac{1}{5}, \cdots$$ Laughlin's states

- sources of a_μ field are anyons w. Aharonov-Bohm phases $\frac{\theta}{2\pi} = \frac{1}{k} = 1, \frac{1}{3}, \cdots$

Bosonic description: boundary



$\varphi(u,t), \quad a_i|_{\partial\mathcal{M}} = \partial_i \varphi$ (1+1)d scalar field

Hall current

- boundary d.o.f.'s are dynamic: add Hamiltonian

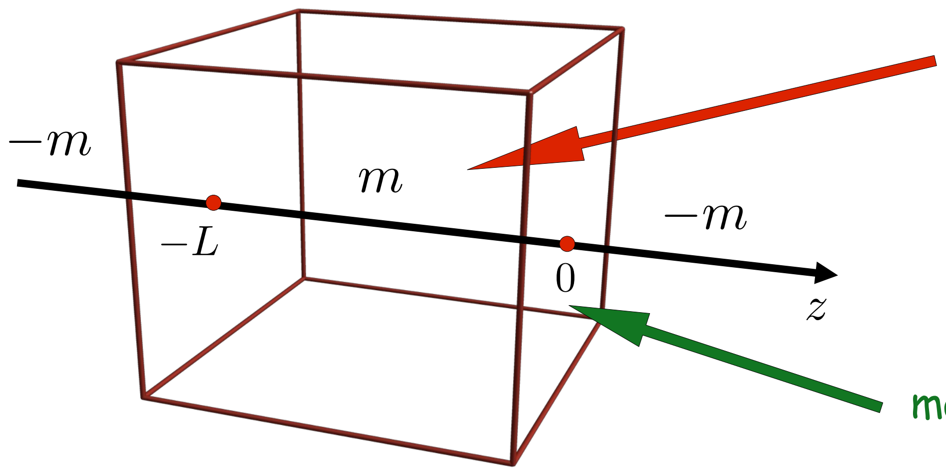
$$S_{\rm bdry}[\varphi] = \frac{k}{4\pi}\int_{\partial\mathcal{M}} dt du \left(\partial_u\varphi\,\dot\varphi - (\partial_u\varphi)^2\right) \qquad \longrightarrow \quad \varphi = \varphi(u+t)$$

- chiral boson, conformal field theory with $(c,\bar c) = (1,0)$; k spans critical line
- chiral anomaly matches the Hall current $\dfrac{dQ_{\rm bdry}}{dt} = \oint dx^i \varepsilon_{ik} J^k$ anomaly inflow
- anyon at the boundary described by CFT vertex operator

$$V_q(u) = e^{iq\varphi(u)} \qquad q = \frac{n}{k}, n \in \mathbb{Z}$$ charge

Exact bosonization of (1+1)d fermion $\psi \equiv e^{i\varphi}$

Topological Insulators in (3+1)d



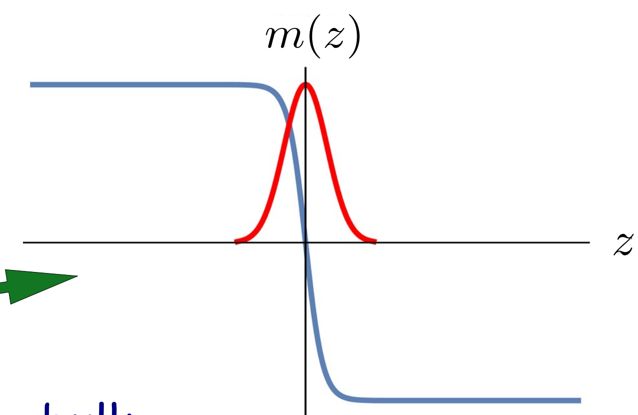
bulk theta term w. $\theta = \pi \sim -\pi$

for Time-Reversal invariance

massless (2+1)d fermion at the surface

- quick fermionic description: take a Dirac fermion in (3+1)d with mass -m
- chiral rotation $\psi \to \exp(i\alpha\gamma^5)\psi$ in $-L < z < 0$ w. $\alpha = -\frac{\pi}{2}$

$$S_{\text{Dirac}}[\psi, -m, A] \to S_{\text{Dirac}}[\psi, m, A] + \frac{\pi}{8\pi^2}\int F^2, \quad F = dA$$



- theta term in the low-energy limit

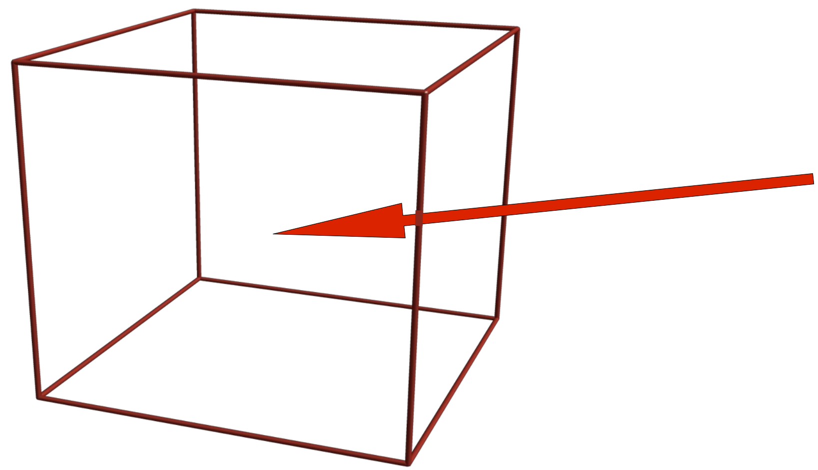
→ bulk: Topological Insulator

→ surface: massless (2+1)d fermion

- parity anomaly $S_{\text{bdry}}[A] = \pm\frac{1}{8\pi}\int_{\partial\mathcal{M}} AdA$ compensated by bulk

Bosonic description: bulk

(Cho, Moore, '11)



$$J^{\mu} = \frac{1}{2\pi}\varepsilon^{\mu\nu\rho\sigma}\partial_{\nu}b_{\rho\sigma}$$ matter current

$$J^{\mu\nu} = \frac{1}{2\pi}\varepsilon^{\mu\nu\rho\sigma}\partial_{r}a_{\sigma}$$ vortex current

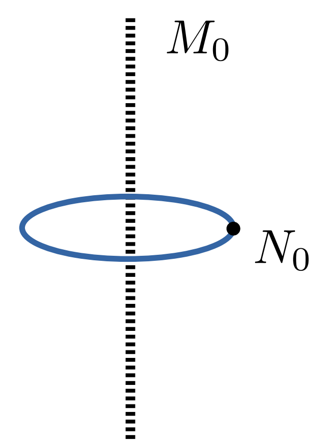
a_{μ} one-form, $b_{\mu\nu}$ two-form hydro fields

$$S_{\rm bulk}[a,b,A] = \int_{\mathcal{M}} \frac{k}{2\pi}bda + \frac{1}{2\pi}bdA + \frac{\pi}{8\pi^2}dada \quad \Rightarrow \quad S_{\rm bulk}[A] = \frac{1}{8\pi k^2}\int F^2$$

- $k=1$ theory reproduces fermionic theta term
- $k>1$ describes the braiding of particles and vortex lines in 3d

$$\frac{\theta}{2\pi} = \frac{N_0 M_0}{k}$$

$$N_0, M_0 \in \mathbb{Z}$$



Bosonic description: (2+1)d boundary

$$S_{\rm bulk}[a,b,A] = \int_{\mathcal{M}} \frac{k}{2\pi} bda + \frac{1}{2\pi} bdA + \frac{1}{8\pi} dada$$

- Gauge invariance ➡ surface term $\quad b|_{\partial\mathcal{M}} = d\zeta, \qquad a|_{\partial\mathcal{M}} = a$

$$S_{\rm surf}[a,\zeta,A=0] = \frac{k}{2\pi}\int_{\partial\mathcal{M}} \zeta da = \frac{k}{2\pi}\int_{\partial\mathcal{M}} d^3x\, \varepsilon^{ij}\zeta_i \dot{a}_j, \qquad (a_0 = \zeta_0 = 0)$$

- add dynamics: nonlocal Abelian gauge theory

$$S_{\rm surf} = \int_{\partial\mathcal{M}} \frac{k}{2\pi}\zeta da + \frac{1}{2\pi}\zeta dA + \frac{g}{16\pi^3}\int_{\partial\mathcal{M}} f_{\mu\nu}(x)\frac{1}{(x-y)^2} f_{\mu\nu}(y), \qquad f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$$

- $S_{\rm surf}[A]$ reproduces the fermionic effective action to $O(A^2)$

➡ $k=1$ bosonization of massless fermions in (2+1)d (semiclassical approx.)

➡ $k>1$ bosonization of anyons

Properties of nonlocal Abelian theory

$$S_{\rm surf}[a,\zeta,A] = \int_{\partial\mathcal{M}} \frac{1}{2\pi}\zeta dA + \frac{k}{2\pi}\zeta da + \frac{g}{4\pi} a_\mu \left(\frac{-\delta_{\mu\nu}\partial^2 + \partial_\mu\partial_\nu}{\sqrt{-\partial^2}}\right) a_\nu,$$

$$\left(\frac{1}{\sqrt{-\partial^2}}\right)_{x,y} \sim \frac{1}{(x-y)^2}$$

$$S_{\rm surf}[\zeta,A] = \int_{\partial\mathcal{M}} \frac{1}{2\pi}\zeta dA + \frac{k^2}{4\pi g}\zeta_\mu \left(\frac{-\delta_{\mu\nu}\partial^2 + \partial_\mu\partial_\nu}{\sqrt{-\partial^2}}\right) \zeta_\nu$$

MAGIC (Fradkin, Kivelson, '96)

- electric and magnetic monopoles corresponding to bulk excitations

  electric $\frac{1}{2\pi}\int_{S^2} d\zeta = \frac{N_0}{k},$ magnetic $\frac{1}{2\pi}\int_{S^2} da = M_0$

- critical line $g > g_c$ where neither monopole condenses (NOT as compact YM)
- it matches QED in d=3&4 (Hsiao, Son, '17) for large number of fermions

  $$N_f \to \infty, \quad \lambda = e^2 N_f \text{ finite}, \quad \lambda \propto g$$ (semiclassical limit)

- explicit electric-magnetic (particle-vortex) duality (Geraedts, Motrunich '12)

$$S_{\rm surf}[a,\zeta,C] + \frac{1}{2\pi}\int C dA \longrightarrow \widetilde{S}_{\rm surf} = S_{\rm surf}[\zeta,a,A] \qquad \widetilde{g} \leftrightarrow \frac{1}{g}, \qquad a \leftrightarrow \zeta$$

Quantization by adding one dimension

- theory becomes local once the gauge interaction is rewritten in (3+1)d

$$S[a,\zeta] = \int d^3x dz \left[\frac{k}{2\pi}\zeta da\right]\delta(z) - \frac{g}{16\pi}\left(\partial_\mu \hat{a}_\nu - \partial_\nu \hat{a}_\mu\right)^2 , \qquad \hat{a}_\mu(z=0) = a_\mu \qquad x^\mu = (x^\alpha, z)$$

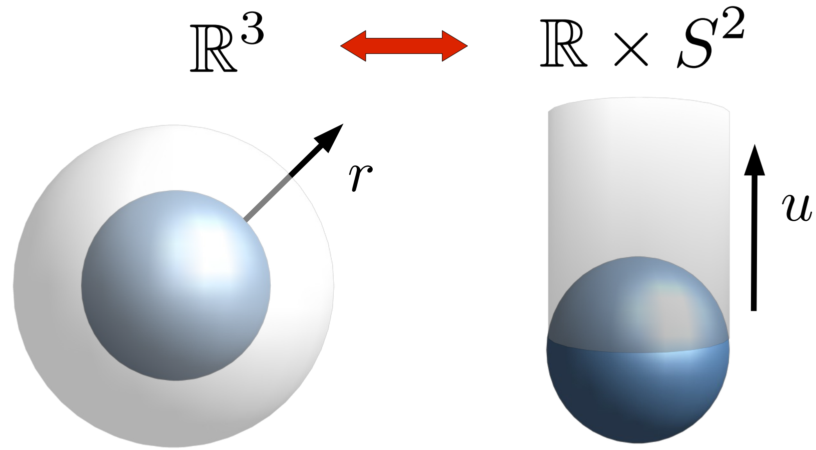
$$= \frac{k^2}{4\pi g}\int_{\partial\mathcal{M}} \zeta_\alpha \left(\frac{-\delta_{\alpha\beta} - \partial_\alpha\partial_\beta}{\sqrt{-\partial^2}}\right)\zeta_\beta , \qquad \left(\frac{1}{\partial^2}\right)_{4D} = \frac{1}{x^2+z^2} \to \frac{1}{x^2} = \left(\frac{1}{\sqrt{\partial^2}}\right)_{3D}$$

- do canonical quantization: solitonic and oscillator modes

$$Z = \sum_{N_0, M_0 \in \mathbb{Z}} e^{-S[\hat{a}_{\rm sol}, \zeta_{\rm sol}]} \int_{N_0, M_0} \mathcal{D}\hat{a}\mathcal{D}\zeta \; e^{-S[\hat{a}_{\rm osc}, \zeta_{\rm osc}]}$$

- classical solutions of magnetic (a_μ) and electric (ζ_μ) monopoles in (3+1)d
- determinant of oscillator modes evaluated by zeta-function regularization
- partition function computed for torus $\mathbb{T}^3$ and $S^1 \times S^2$ geometries

Partition function on $S^1 \times S^2$ and CFT



- Conformal map $r = R\exp(u/R)$
- r dilatations match u time translations

$$\mathcal{E}_{N_0,M_0} = \frac{1}{R}\Delta_{N_0,M_0}$$ scale dimensions

$$Z = \mathrm{Tr} e^{-\beta H} = \sum_{N_0,M_0\in\mathbb{Z}} \exp\left[-\frac{\beta\lambda}{R}\left(\frac{N_0^2}{g} + gM_0^2\right)\right] \prod_{\ell=1}^{\infty}\left[1-\exp\left(-\frac{\beta}{R}\ell\right)\right]^{-2\ell}$$

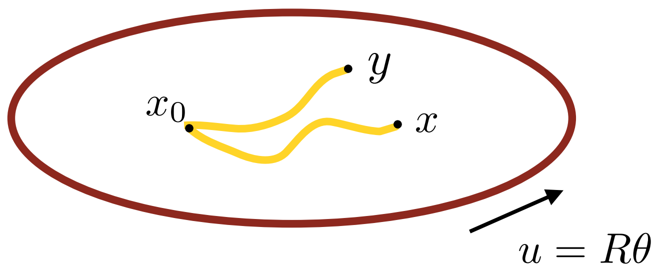
- Spectrum is manifestly self-dual: $N_0 \longleftrightarrow M_0, \quad g \longleftrightarrow 1/g,$
- checks of conformal invariance at the quantum level:

  - no Casimir effect in the energy spectrum ⟷ no conformal anomaly in (2+1) d

  - integer-spaced dimensions of descendant (derivative) fields

Bosonization in topological states

Bulk theory tell us how to bosonize boundary fermions: ex. (1+1)d



$$S_{\text{bulk}}[a] = \frac{k}{4\pi} \int ada$$

- introduce open Wilson loops

$$W[\gamma(x, x_o)] = \exp\left(iq \int_{x_0}^{x} dx^\mu a_\mu \right) \sim W(x)$$

- their bulk correlator is just the A-B phase

$$\langle W_1(x) W_2(y) \rangle \sim \exp\left(i2\pi \frac{q_1 q_2}{k} \right), \quad (x-y) \to e^{i2\pi}(x-y)$$

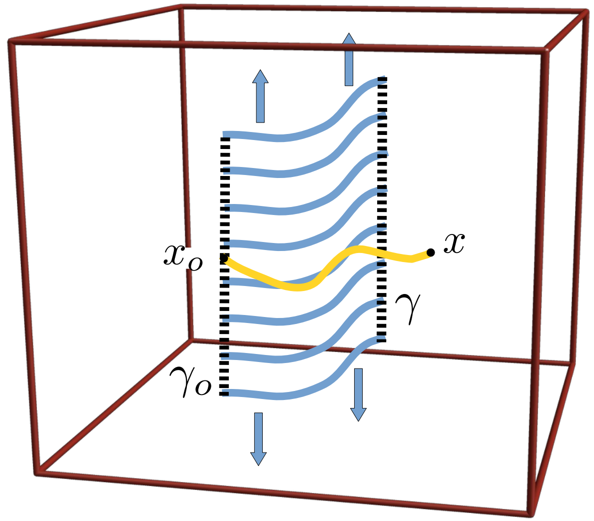
- bring point to the edge: flat connection determined by boundary field $a = d\varphi$

$$W(x)|_{\partial\mathcal{M}} = e^{iq\varphi(u)} = V_q(u)$$

- edge dynamics implies a nontrivial edge correlator $\langle V_{q_1}(u_1) V_{q_2}(u_2) \rangle = (x_1 - x_2)^{q_1 q_2/k}$

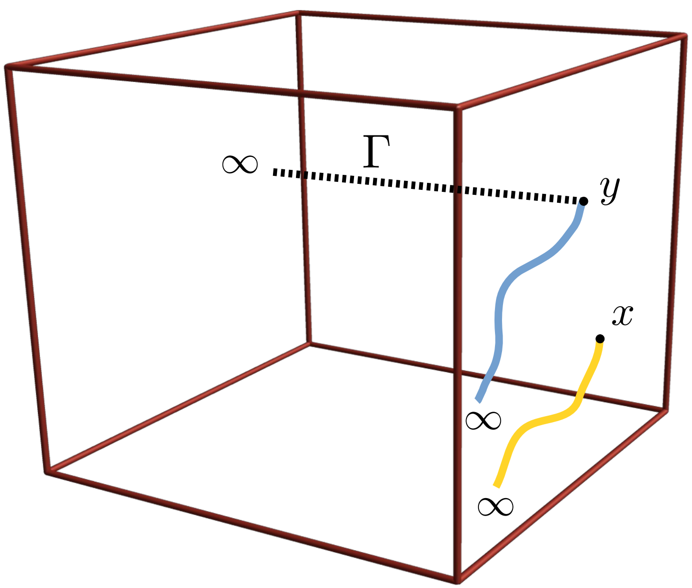
→ chiral fermion field is represented by end point of Wilson loop

Bosonization in (2+1)d



$$W[\gamma(x, x_o)] = \exp\left(iq\int_{x_o}^{x} dx^\mu a_\mu\right)$$ bulk Wilson line

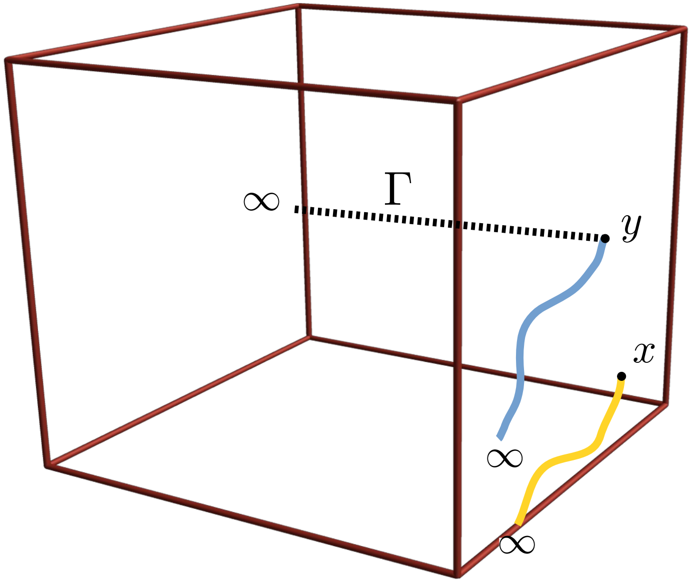
$$Y[\Sigma(\gamma,\gamma_o)] = \exp\left(ip\int_{\gamma_o}^{\gamma} d\Sigma^{\mu\nu}\, b_{\mu\nu}\right)$$ bulk Wilson surface



- bring operators to boundary
- vertex line hits the surface
- $b|_{\partial\mathcal{M}} = d\zeta, \qquad a|_{\partial\mathcal{M}} = a$

$$Y[\Sigma] \to \exp\left(ip\int^{y}\zeta - ip\int_{\Gamma}^{y}\zeta\right) = \widetilde{W}[\gamma(y)] \times \text{(TAIL)}$$

- the vortex line remains in the bulk as a "topological tail"



Wilson loop/surface brought to boundary

$$W(x) = \exp\left(iN_0 \int_\infty^x a\right)$$

$$Y[y,\Gamma] = \exp\left(iM_0 \int_\infty^y \zeta\right) \exp\left(-iM_0 \int_\Gamma^y \zeta\right) = \widetilde{W}(y) \times \text{(TAIL)}$$

- they create (topological) charges: $W[x]$ electric, $\widetilde{W}[y]$ magnetic

$$\hat{Q}_e W(x) = N_0 W(x), \qquad \hat{Q}_m \widetilde{W}(y) = M_0 \widetilde{W}(y), \qquad Q_e = \frac{1}{2\pi}\int_\Sigma \varepsilon^{ij}\partial_i\zeta_j, \qquad Q_m = \frac{1}{2\pi}\int_\Sigma \varepsilon^{ij}\partial_i a_j$$

- the two-component spinor is obtained by multiplying these loop operators

$$\Psi(x) = \begin{pmatrix} \exp(i\int^x a_+) \\ \exp(i\int^x a_-) \end{pmatrix}, \qquad a_\pm = \frac{a}{2} \pm \zeta, \quad k = N_0 = M_0 = 1$$

cf. Ising model

$\sigma\Psi \sim \mu$

with correct T, P transformations

- (2+1)d nonlocal dynamics gives power-law correlator for $\langle \Psi(x)\bar{\Psi}(y)\rangle$
- bosonization dictionary in progress (AC, Maffi, in preparation)

Remarks on (2+1)d bosonizaton:

- bulk vortex line attached to fermion (anyon) does not affect the dynamics but is there → bosonic description needs one extra dimension

  Hints:

  - vortex lines in superconductors are known to be "fermionic"

  - modern view of anomalies requires a topological theory in one extra dimension

Conclusions

- the non-local Abelian gauge theory provides an interesting semiclassical bosonic description of massless free and interacting fermions in (2+1)d

  → critical g line, manifest self-duality, conformal invariance

- bosonization in higher dimensions can be understood using the bulk-boundary correspondence of topological states:

  - needs higher-rank gauge fields and (d+1) bulk topological actions

  - bosonization of (3+1)d fermions: chiral anomaly from hydrodynamics, (4+1)d TI, geometrical effective theory

(Abanov, Wiegmann; Monteiro, Abanov, Nair)