Universal Transport Properties in the Quantum Hall Effect

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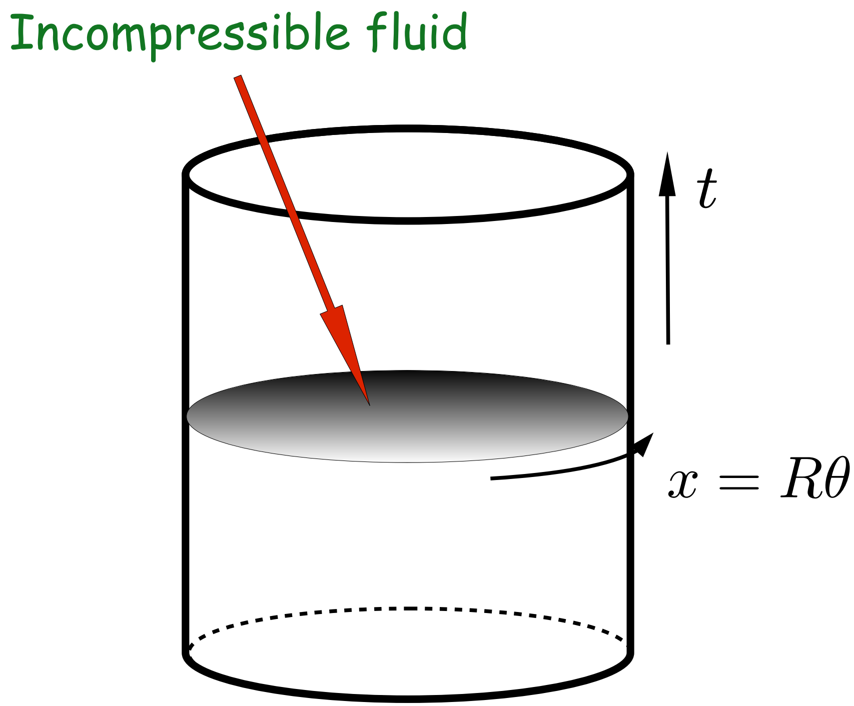
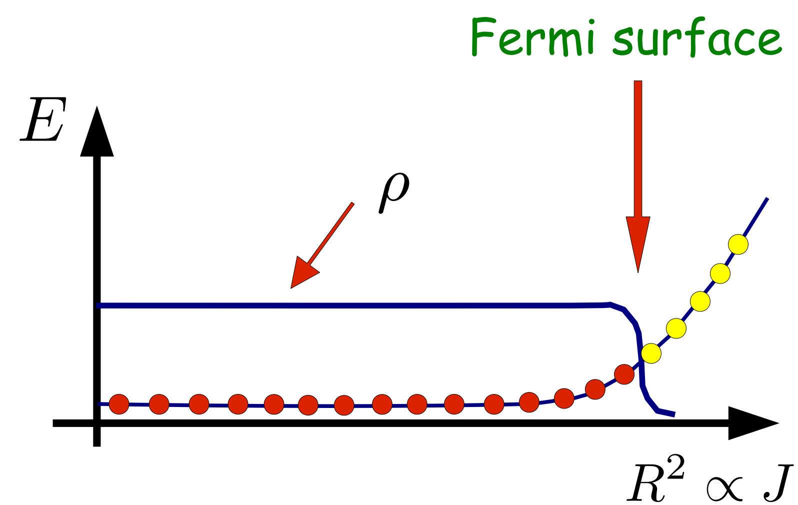
Outline

- Chern-Simons effective action: bulk and edge
- Transport due to chiral and gravitational anomalies
- Wen-Zee term, 'orbital spin' and Hall viscosity
- Orbital spin at the edge and its universality

Edge excitations





edge ~ Fermi surface: linearize energy $\varepsilon(k) - \varepsilon_F = vk = \frac{v}{R}n, \quad n \in \mathbb{Z}$

relativistic field theory in (1+1) dimensions with chiral excitations (X.G.Wen, '89)

- Weyl fermion (non interacting) $\nu = 1$
- Interacting fermion $\nu = \frac{1}{k}$ chiral boson (Luttinger liquid)

Chern-Simons action & Hall Current

$$S\left[A\right]=\frac{\nu}{4\pi}\int AdA=\frac{\nu}{4\pi}\int\varepsilon^{\mu\nu\rho}A_{\mu}\partial_{\nu}A_{\rho}$$ Laughlin state $\nu=\frac{1}{k}=1,\frac{1}{3},\ldots$

$$\rho=\frac{\delta S}{\delta A_0}=\frac{\nu}{2\pi}\mathcal{B}$$ Density $$J^{i}=\frac{\delta S}{\delta A_i}=\frac{\nu}{2\pi}\varepsilon^{ij}\mathcal{E}^{j}$$ Hall current

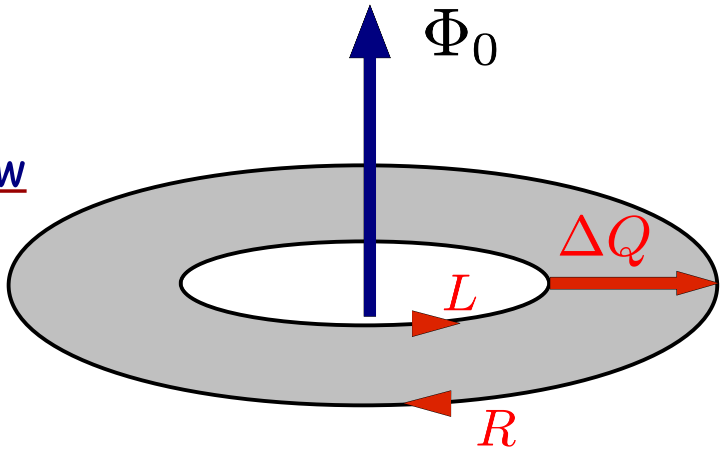
Introduce Wen's hydrodynamic matter field a_μ and current $j^{\mu}=\frac{1}{2\pi}\varepsilon^{\mu\nu\rho}\partial_{\nu}a_{\rho}$

$$S\left[a,A\right]=\int-\frac{1}{4\pi\nu}ada+A\cdot j \quad \Rightarrow \quad S[A]=\frac{\nu}{4\pi}\int AdA$$

- Hall current is topological
- Sources of a_μ field are anyons
- Needs boundary action $S_b\left[\varphi\right],\quad a_\mu|_b=\partial_\mu\varphi$ $\Rightarrow$ chiral boson CFT
- Bulk topological theory is tantamount to conformal field theory on boundary

Boundary CFT and chiral anomaly

- edge states are chiral fermions/bosons
- chiral anomaly: boundary charge is not conserved
- bulk (B) and boundary (b) compensate: anomaly inflow



$$\partial_i J^i + \partial_t \rho = 0, \ \rightarrow \ \oint dx J_B + \partial_t Q_b = 0$$

- adiabatic flux insertion (Laughlin)

$$\Phi(t): \ 0 \rightarrow \ n\,\Phi_0, \qquad H[\Phi + n\,\Phi_0] = H[\Phi], \quad n = 1, 2, \ldots$$ spectral flow

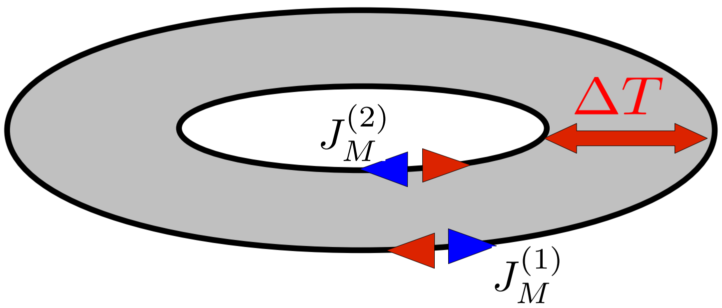
$$Q_R \rightarrow Q_R + \Delta Q_b, \qquad \Delta Q_b = \int_{-\infty}^{+\infty} dt \oint dx \ \partial_t \rho = \frac{\nu}{2\pi} \int \mathcal{F} = \nu\, n$$

- edge chiral anomaly: → exact quantization of the Hall current
  → universal transport coefficient $\sigma_H = \frac{\nu}{2\pi}$
  → bulk-boundary correspondence

Thermal current and gravitational anomaly

- gravitational anomaly in the chiral edge theory $(c, \bar{c})\,,\quad c \neq \bar{c}$

$$D^z \mathcal{T}_{zz} = -\frac{c}{24} D_z \mathcal{R}, \quad z = x^1 + i\, x^2, \quad (+\,\text{h.c.}\ \ \text{for}\ \bar{c})$$



- Casimir energy and 'Casimir matter current'

$$E_0(T) \sim \langle \mathcal{T}_{zz} + \mathcal{T}_{\bar{z}\bar{z}} \rangle_T, \qquad J_M(T) \sim \langle \mathcal{T}_{zz} - \mathcal{T}_{\bar{z}\bar{z}} \rangle_T,$$

- $\frac{d}{dT}$ → specific heat and thermal current
- unbalanced edges $\Delta J_M = J_M^{(1)} - J_M^{(2)} = \kappa\, \Delta T$

$$\boxed{\kappa = \frac{\pi T}{6}(c - \bar{c})}$$

(Read, Green '00; A.C., Huerta, Zemba '02)

- It has been measured (Heiblum et al. '14-'19; Wang et al. '18)
- bulk-boundary correspondence: gravitational Chern-Simons action

$$S_{\text{grav}}[g] = \frac{c - \bar{c}}{96\pi} \int \text{Tr}\left(\Gamma d\Gamma + \frac{2}{3}\Gamma^3\right),$$

$g_{\mu\nu}$ metric background in (2+1)-d $\mu, \nu = 0, 1, 2$

(Stone '12; Gromov et al. '15)

Wen-Zee-Fröhlich action

- consider spatial metric g_{ij} only and corresponding $O(2)$ spin connection ω_μ

$$g_{ij} = e_i^a e_j^a, \quad \omega_\mu^{ab} = \omega_\mu(e)\varepsilon^{ab}, \quad i,j,a,b = 1,2, \qquad \delta g_{ij} = \partial_i u_j + \partial_j u_i \quad \text{strain}$$

$$S_{WZ}\left[a, A, g\right] = \frac{1}{2\pi}\int -\frac{1}{2\nu} a da + j\cdot(A + s\,\omega) \rightarrow S_{WZ}\left[A, g\right] = \frac{\nu}{4\pi}\int AdA + 2s\,Ad\omega + s^2\,\omega d\omega$$

$$\rho = \frac{\delta S}{A_0} = \frac{\nu}{2\pi}\left(B + \frac{s}{2}\mathcal{R}\right)$$

Wen-Zee shift

$$N = \nu N_\phi + \nu s\,\chi$$

$$T_{ij} = -2\frac{\delta S}{\delta g^{ij}} = \frac{\eta_H}{2}\varepsilon_{ik}\dot{g}_{jk} + (i \leftrightarrow j)$$

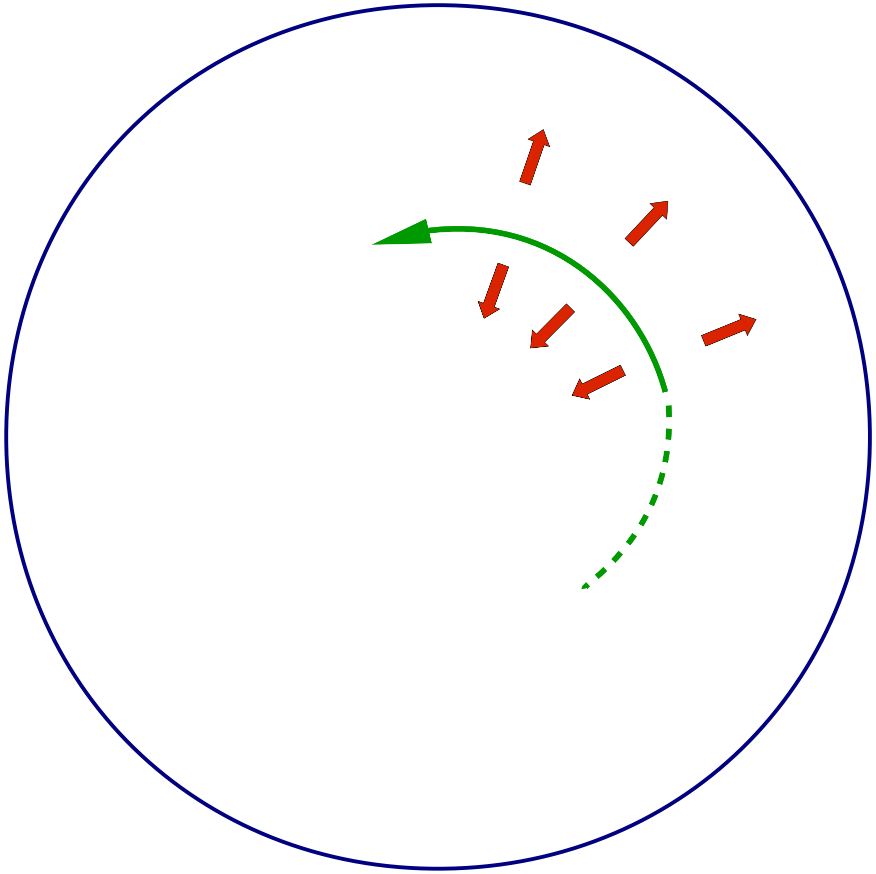
Hall viscosity

$$\eta_H = \frac{\rho_0 s}{2}$$

- η_H further transport coefficient (Avron et al., Read et al.)
- s electron 'orbital spin', e.g. $s = n + \frac{1}{2}$ on n-th Landau level

Hall viscosity

$$T_{ij} = \frac{\eta_H}{2} \varepsilon_{(ik} \dot{g}_{j)k}$$



- Constant stirring creates an orthogonal static force, non dissipative

Is Hall viscosity universal?

- If YES:

  - further universal 'geometric' transport coefficient $\eta_H \propto s$ orbital spin

  - suggests that 'composite fermion' excitations are extended objects

  → renewed interest, following Haldane, Read, D.T. Son, Wiegmann,...

- BUT: s is not related to an anomaly!

- how to understand?

  A: study quantities related to s in the edge CFT and check universality

Bulk-boundary correspondence for s

- add boundary terms to the Wen-Zee action (Gromov, Jensen, Abanov '16)

$$S_{WZ} = \frac{\nu}{4\pi}\int_{\mathcal{M}} \left(2sA + s^2\omega\right) d\omega + \frac{\nu}{4\pi}\int_{\partial\mathcal{M}} \left(2sA + s^2\omega\right) K$$

$K = K_i dx^i$ extrinsic curvature needed for Euler characteristic $\chi = 2 - 2g - b$

- predicts ground-state values for charge and spin in the edge CFT

$$Q_0 = \nu\, s \qquad \mathcal{S}_0 = \frac{\nu s^2}{2}$$

- BUT:

  - g-s values are not universal, can be tuned to zero (~ local terms in the 2d action)

  - which boundary conditions?

  - bulk non-universality: this action suggests that islands with different s values can be made in the bulk without closing the gap

Orbital spin at the edge

(AC, Maffi `18)

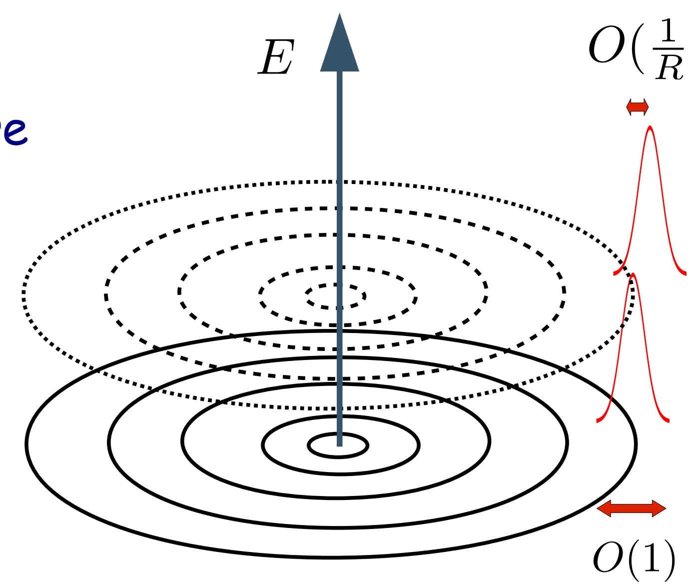
- Explicitly study of Landau levels for integer filling $\nu = n, \ \left(s = n + \frac{1}{2}\right)$
- limit to the edge: momentum $m = O(L)$ and radius $r = O(R), \quad L = R^2 \to \infty$
- wavefunctions of level n are gaussian-localized at $r = R + x$ with spread $\Delta x = O(1)$

$$\psi_{n,L+k}(R+x,\theta) \sim \frac{1}{\sqrt{R}} H_n\left(x - \frac{k+n}{2R}\right) \exp\left[-\left(x - \frac{k+n}{2R}\right)^2\right]$$

- shift outward by $\delta x = n/2R$ is absent for straight boundary (extrinsic curvature)

→ orbital spin $s = n + s_o$ i.e. up to const.

- wavefunctions pushed up in confining potential acquire higher energy for higher Landau levels



Fermion CFT at the edge

- n-th level branch is displaced by $O(n/R)$

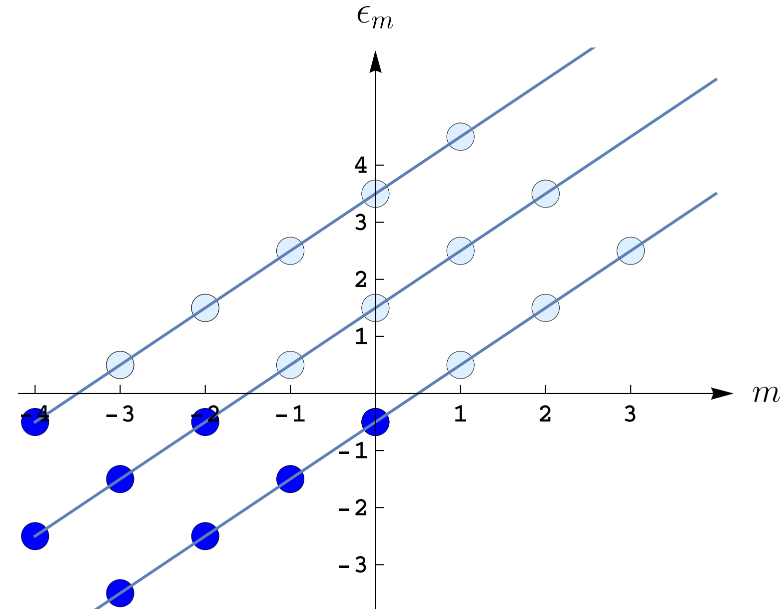
$$H^{(n)} = \frac{v}{R} \sum_{k_n \in \mathbb{Z}} (k_n - \mu_n) : a^{(n)\dagger}_{k_n} a^{(n)}_{k_n} :$$

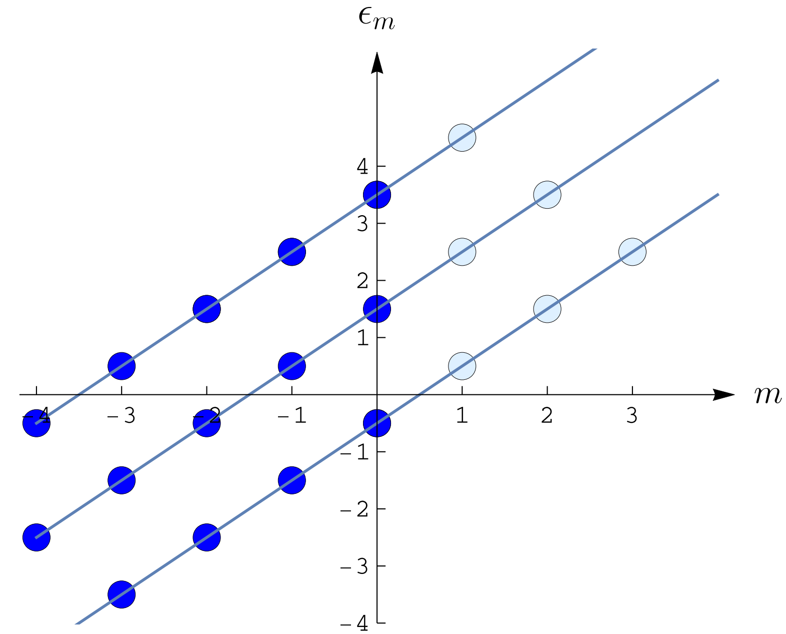
- Can be accounted for by chemical potential shifts $\mu_n = -n + \mu_o$

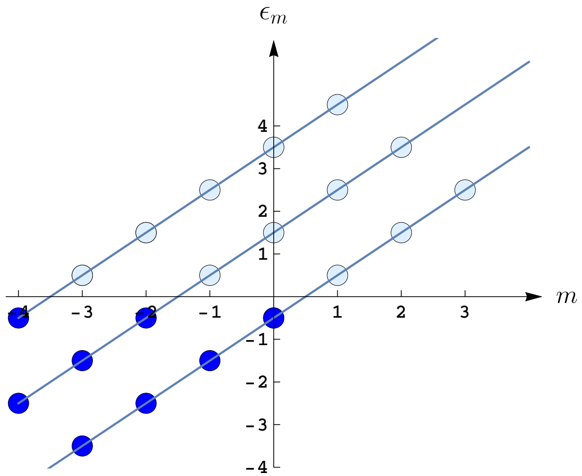
- how to fill the Fermi sea?

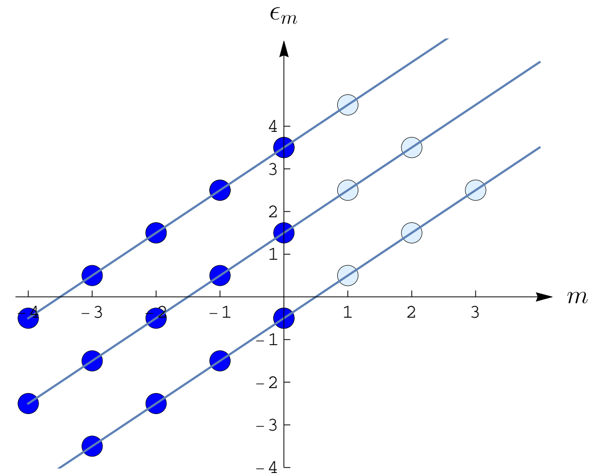
  - top: least energy (smooth boundary)

  - bottom: same momentum (sharp boundary)









- analyze the cases:

  i) single branch

  ➡ no physical effect of orbital spin s

  ii) left: independent (non-interacting) branches, as e.g. integer Hall effect, or connected to a reservoir ($\mu_n = eV_o, \ \forall n$)

  ➡ no effect of s

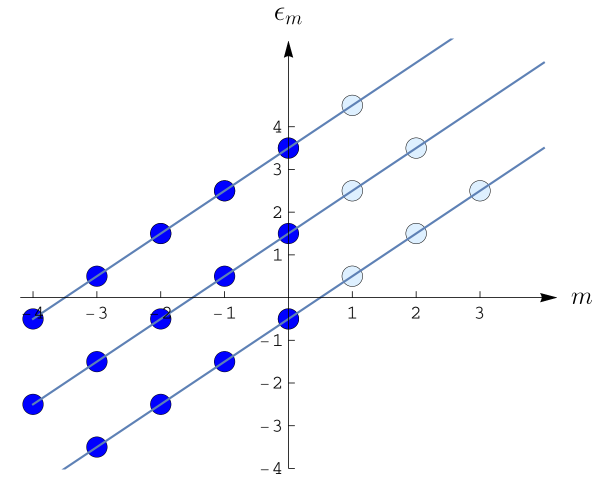
  iii) right: overlapping (interacting) branches, e.g. sharp boundary & hierarchical Hall states, for isolated Hall droplets

  ➡ differences $s_i - s_i \in \mathbb{Z}$ are observable and universal

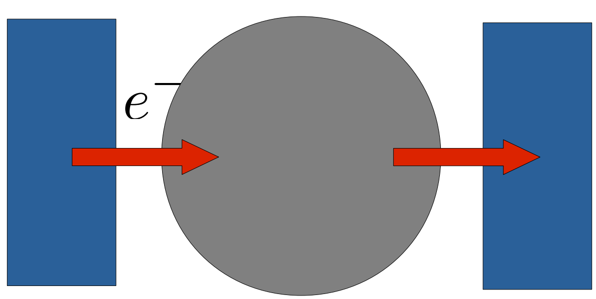
- Excited state w.r.t. standard CFT ground state

$$Q_0 = \sum_i i = \nu \overline{s} + \text{const.}$$

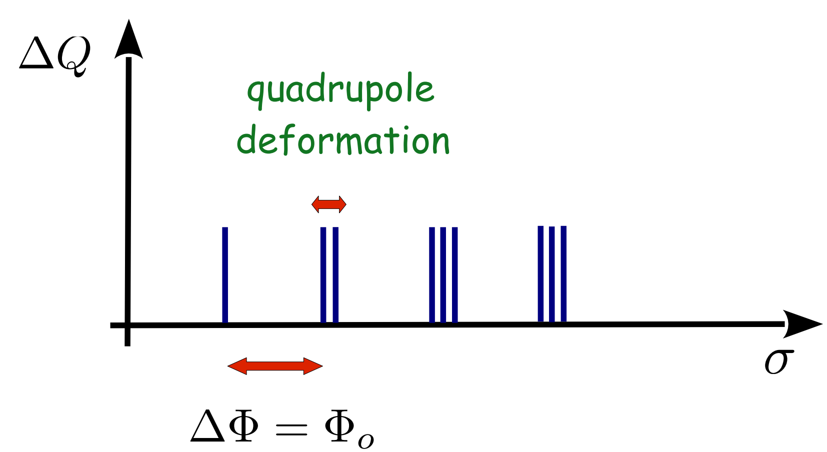
$$\mathcal{S}_0 = \sum_i \frac{i^2}{2} = \frac{\nu \overline{s^2}}{2} + \text{const.}$$



- agreement with WZ action (up to const.)
- orbital spins identified up to const. $s_i \to s_i + s_o$
- Results extend to fractional (hierarchical) filling in the bosonic CFT
- How to measure?
- By Coulomb blockade (tunneling in a isolated droplet at zero bias)



Squeeze area by σ



Conclusions

- charge and heat currents are associated to anomalies of the edge theory

  → universal transport coefficients

- Hall viscosity, proportional to average orbital spin $\bar{s}$ is not universal
- differences of orbital spin $s_i - s_i \in \mathbb{Z}$ are universal and can be observed with edge physics in isolated Hall droplets
- possible experiment: Coulomb blockade in the isolated droplet with quadrupole deformation

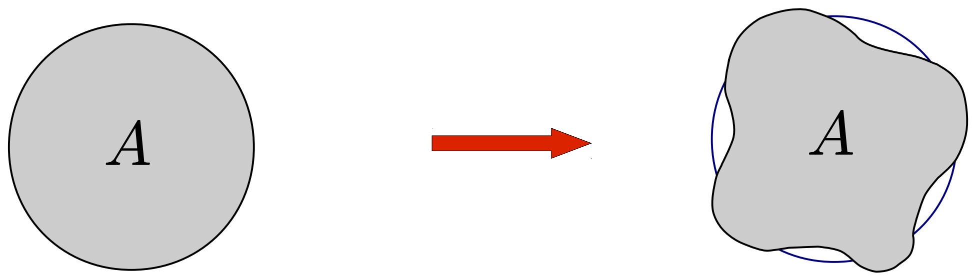
Not discussed

- extension of analysis to fractional fillings by using area-preserving diffeomorphisms (W-infinity algebra)
- other physical effects of orbital spin for QHE on a conical surface (metric singularities are sources of s) (Laskin, Chiu, Can, Wiegmann '16-'18)

Quantum incompressible fluids

- Area-preserving diffeomorphisms of incompressible fluids

$$\int d^2x \ \rho(x) = N = \rho_o A$$ → A = constant



- Fluctuations of the fluid are described by diffs, generated by Poisson brackets

classical $\delta z = \{z, w\}_P \ , \ \delta \bar{z} = \{\bar{z}, w\}_P$ → $\delta\rho(z,\bar{z}) = \{\rho, w\}_P$

quantum $\delta\rho(z,\bar{z}) = i\langle\Omega|[\widehat{\rho},\widehat{w}]|\Omega\rangle = \{\rho, w\}_M$ Moyal brackets

→ W_∞ algebra (GMP sin-algebra)

- Fully understood in the edge CFT $z \to e^{i\theta}$ It reproduces/predicts Jain hierarchy

→ W_∞ minimal models (A.C., Trugenberger, Zemba '96)

- Bulk fluctuations in lowest Landau level are non-local:

$$\delta\rho(z,\bar{z}) = i\langle\Omega|\,[\widehat{\rho},\widehat{w}]\,|\Omega\rangle = i\sum_{n=1}^{\infty}\frac{\hbar^n}{B^n n!}\left[\partial_{\bar{z}}^n\rho\,\partial_z^n w \;-\; \partial_z^n\rho\,\partial_{\bar{z}}^n w\right]$$

(Iso, Karabali, Sakita)

- can be expressed in terms of fields of increasing spin, traceless & symmetric

$$\delta\rho = \frac{i}{B}\partial_{\bar{z}}\left(\rho\partial_z w\right) + \frac{i}{2B^2}\partial_{\bar{z}}^2\left(\rho\partial_z^2 w\right) + \cdots + \mathrm{h.c.}$$

$$= i\partial_{\bar{z}}\,a_z + \frac{i}{B}\partial_{\bar{z}}^2\,b_{zz} + \cdots + \mathrm{h.c.}$$

- Recover Wen hydrodynamic field a_μ plus $\frac{1}{B}$ correction $b_{\mu k}$ $(\mu = 0,1,2,\ k=1,2)$

$$a_\mu = (a_0, a_z, a_{\bar{z}}), \qquad b_{\mu k} = (b_{0z}, b_{0\bar{z}}, b_{zz}, b_{\bar{z}\bar{z}}, b_{\bar{z}z}, b_{z\bar{z}})$$

+ gauge symmetry

$$j^\mu = j^\mu_{(1)} + j^\mu_{(2)} + \cdots, \qquad j^\mu_{(1)} = \varepsilon^{\mu\nu\rho}\partial_\nu\,a_\rho, \qquad a_\rho \to a_\rho + \partial_\rho f$$

$$j^\mu_{(2)} = \frac{1}{B}\varepsilon^{\mu\nu\rho}\partial_\nu\partial_k\,b_{\rho k}, \qquad b_{\rho k} \to b_{\rho k} + \partial_\rho v_k$$

- Spin-one and spin-two fields parameterize matter fluctuations